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神経線維伝導速度分布の blind deconvolution

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inverse problems & measurement

- problems of finding and characterizing unknown parameters and structures by indirect measurements.

- cover wide range of engineering applications, such as computed tomography, non-destructive inspection, non-invasive measurements, shape design, and so on.

inverse problems & **measurement**

- process of experimentally obtaining information about the magnitude of a **quantity**

NOTES

1 Measurement implies a **measurement procedure**, based on a theoretical model.

2 In practice, measurement presupposes a calibrated **measuring system**, possibly subsequently verified.

***International Vocabulary of Basic and General Terms in Metrology* (VIM)**

TOMOGRAPHIC METHOD IN MEASUREMENT SYSTEM

Forward problem:

$$f(\boldsymbol{x}) \underset{\text{measurement}}{\longrightarrow} \rho(\boldsymbol{\xi})$$

Inverse problem:

$$\rho(\boldsymbol{\xi}) \underset{\text{reconstruction}}{\longrightarrow} \hat{f}(\boldsymbol{x})$$

Integral Formulation

$$\begin{aligned}\rho(\boldsymbol{\xi}) &= \int_{\Omega} f(\boldsymbol{x})\, d\Omega \\ \rho(\boldsymbol{\xi}) &= \mathcal{R}\{f(\boldsymbol{x})\}\end{aligned}$$

$$\exists | \mathcal{R}^{-1}\{\rho(\boldsymbol{\xi})\} \Rightarrow f(\boldsymbol{x})$$

Well-Conditioned/Posed

1. existence
2. uniqueness
3. smoothness

Example: CT

$$
\begin{aligned}
\rho(\boldsymbol{\xi}) &= \rho(\theta, u) = \int_L f(\boldsymbol{x}) \; ds \\
f(x, y) &= \frac{1}{2\pi^2} \int_0^{\pi} \int_{-R}^{R} \frac{1}{x \cos\theta + y \sin\theta - u} \; \frac{\partial \rho}{\partial u} \; du \; d\theta \\
&= \mathcal{R}^{-1}\{\rho(\theta, u)\}
\end{aligned}
$$

Reconstruction:

$$\operatorname*{argmin}_{f \in \mathcal{H}_1} \|Kf - g\| = \hat{f}$$

$$\Longleftrightarrow K^* K \hat{f} = K^* g \Longleftrightarrow K\hat{f} = Pg \Longleftrightarrow \hat{f} = K^+ g$$

$$K^+ : \text{bounded} \Leftrightarrow \#\{\lambda_n\} < \infty \longrightarrow \text{ill-posed}$$

Regularization:

$$\exists (K + \alpha I)^{-1} : \text{bounded}$$

$$\|Kf - g\|^2 + \alpha \|f\|^2 \to \min$$

$$\|Kf - g\|^2 + R(f; \alpha) \to \min$$

Projection Onto Convex Sets:

$$f_0 \in \mathcal{C}_0 \cap \left(\bigcap_{i=1}^{m} \mathcal{C}_i \right)$$

$$T = P_m P_{m-1} \cdots P_1, \qquad P_i : \mathcal{H} \to \mathcal{C}_i, \quad T^n f \to f_0$$

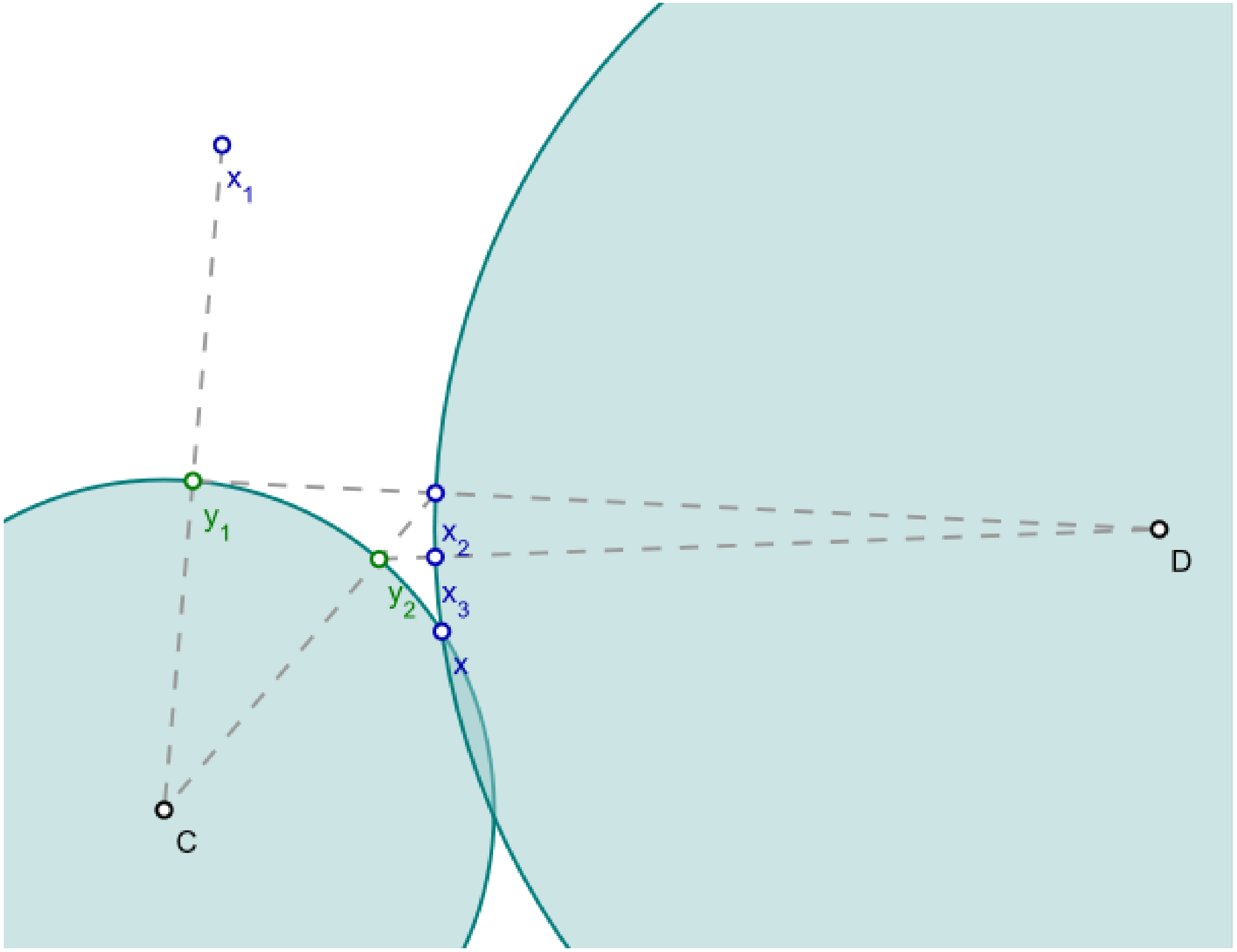
Regularization: 正則化 （最適化問題）

- 正則化パラメータの決定法
  - L-curve 法
  - 最適パラメータ決定理論　　　観測ノイズが必要
  - ベイジアンモデルによるハイパーパラメータ推定(MAP推定)
  - …
  - …

POCS: 凸射影法 （非線形反復法）

- 反復アルゴリズムの収束性
- 制約条件(先験情報)の設定法
- 凸集合の境界に収束　　　制約条件が有効

alternate Projection Onto Convex Sets method



神経線維伝導速度分布の blind deconvolution

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Estimation of the conduction velocity distribution of human sensory nerve fibers

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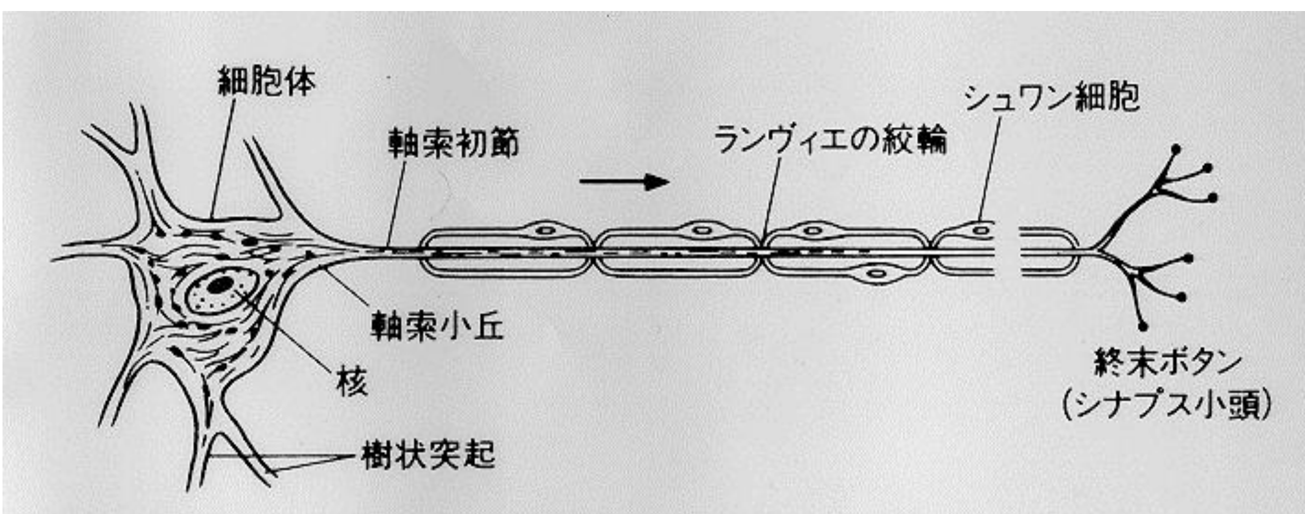
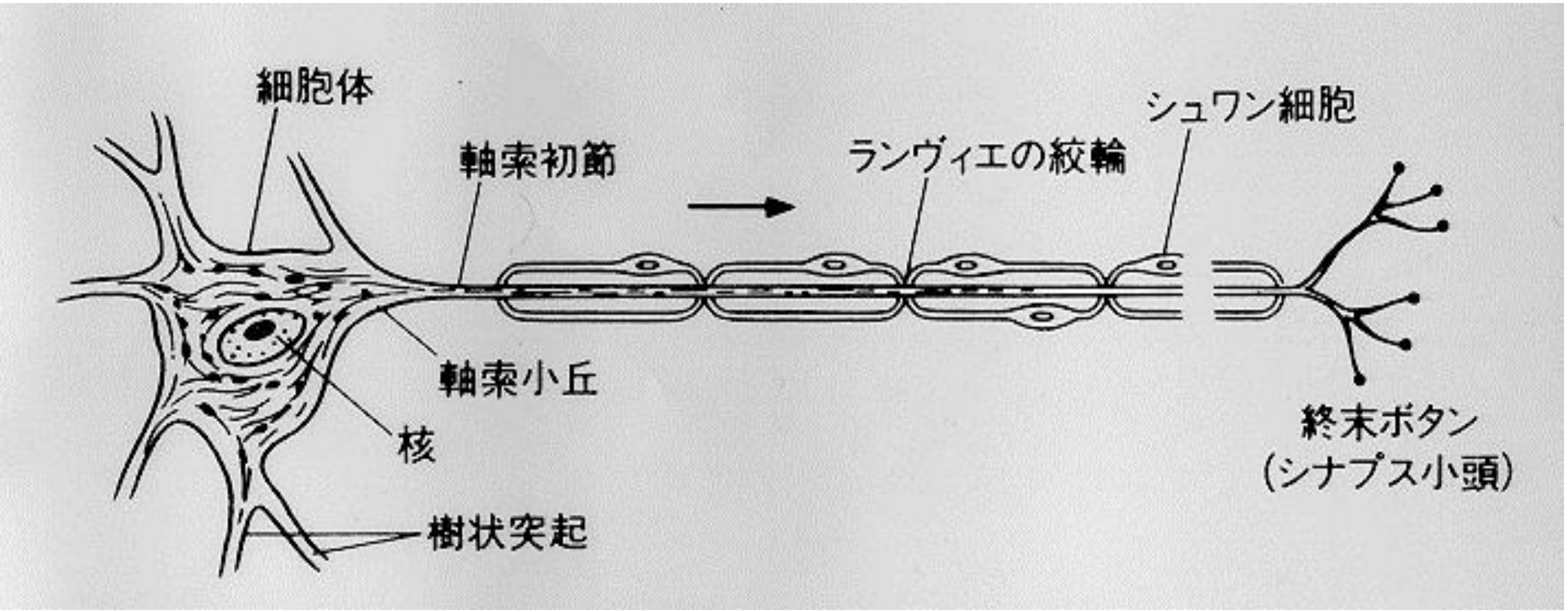
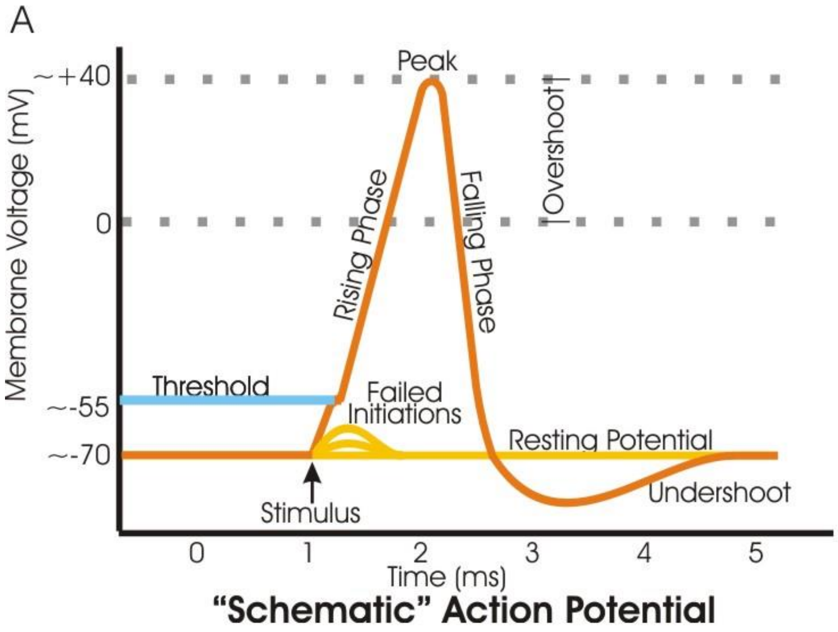
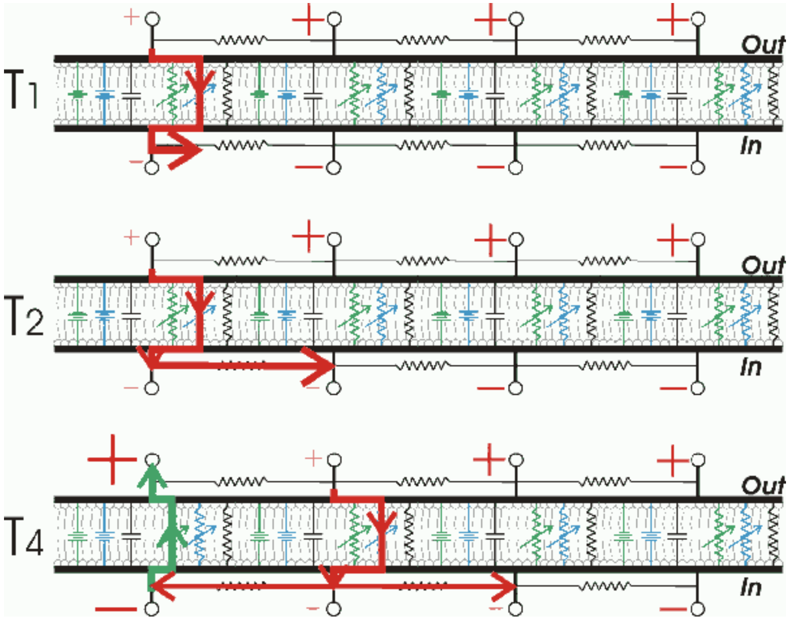
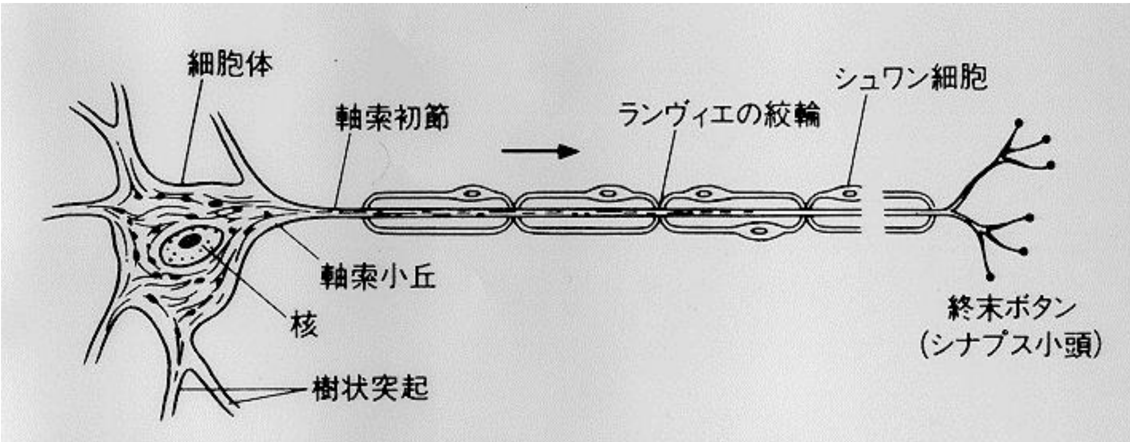


神経線維伝導速度分布の blind deconvolution





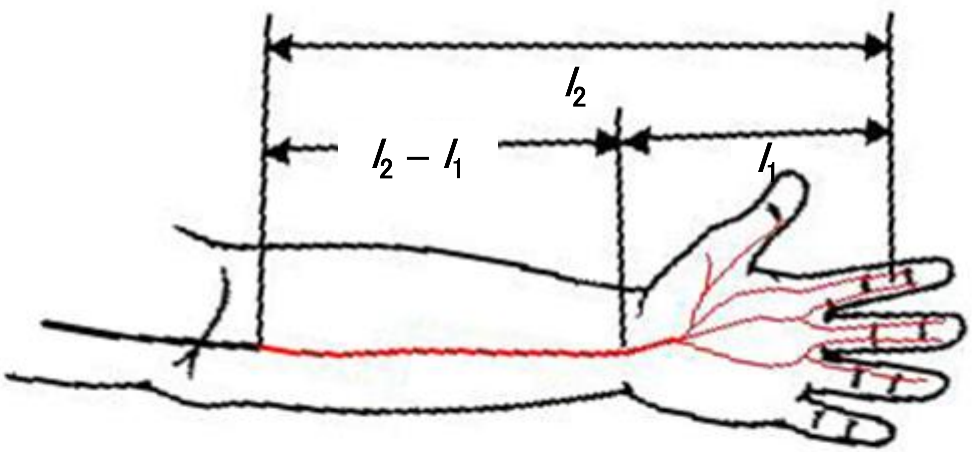


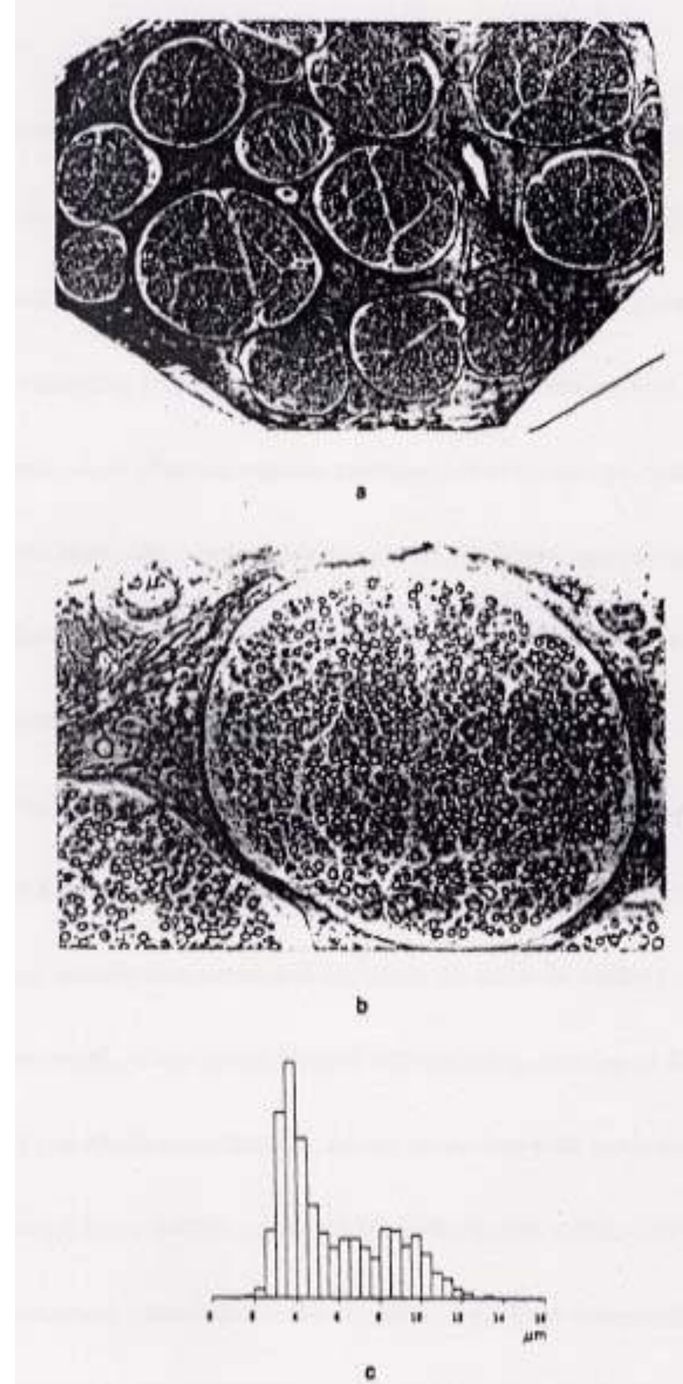


diabetes, heavy-metal poisoning

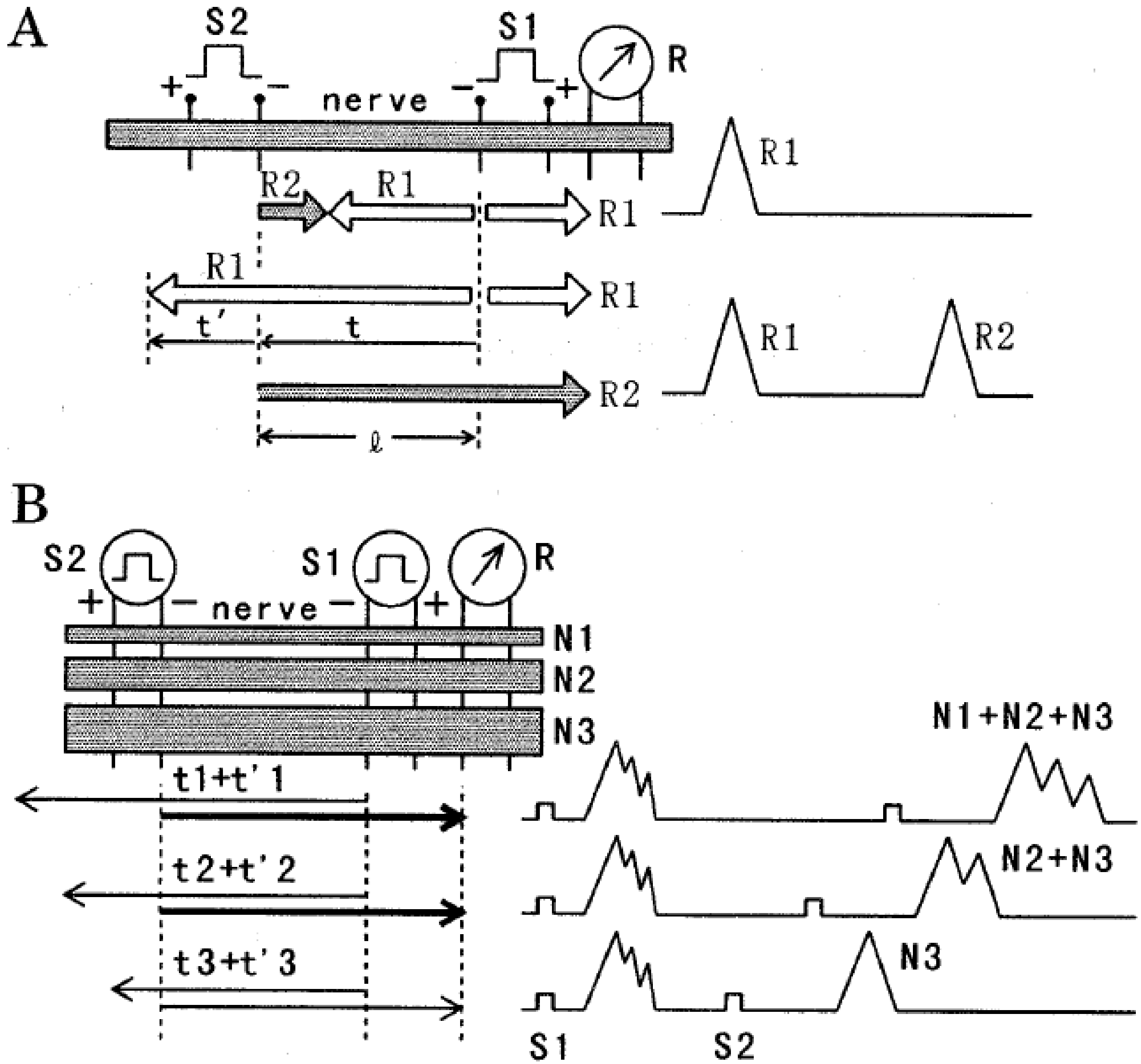
➔ decrease in conduction velocity

amyelination

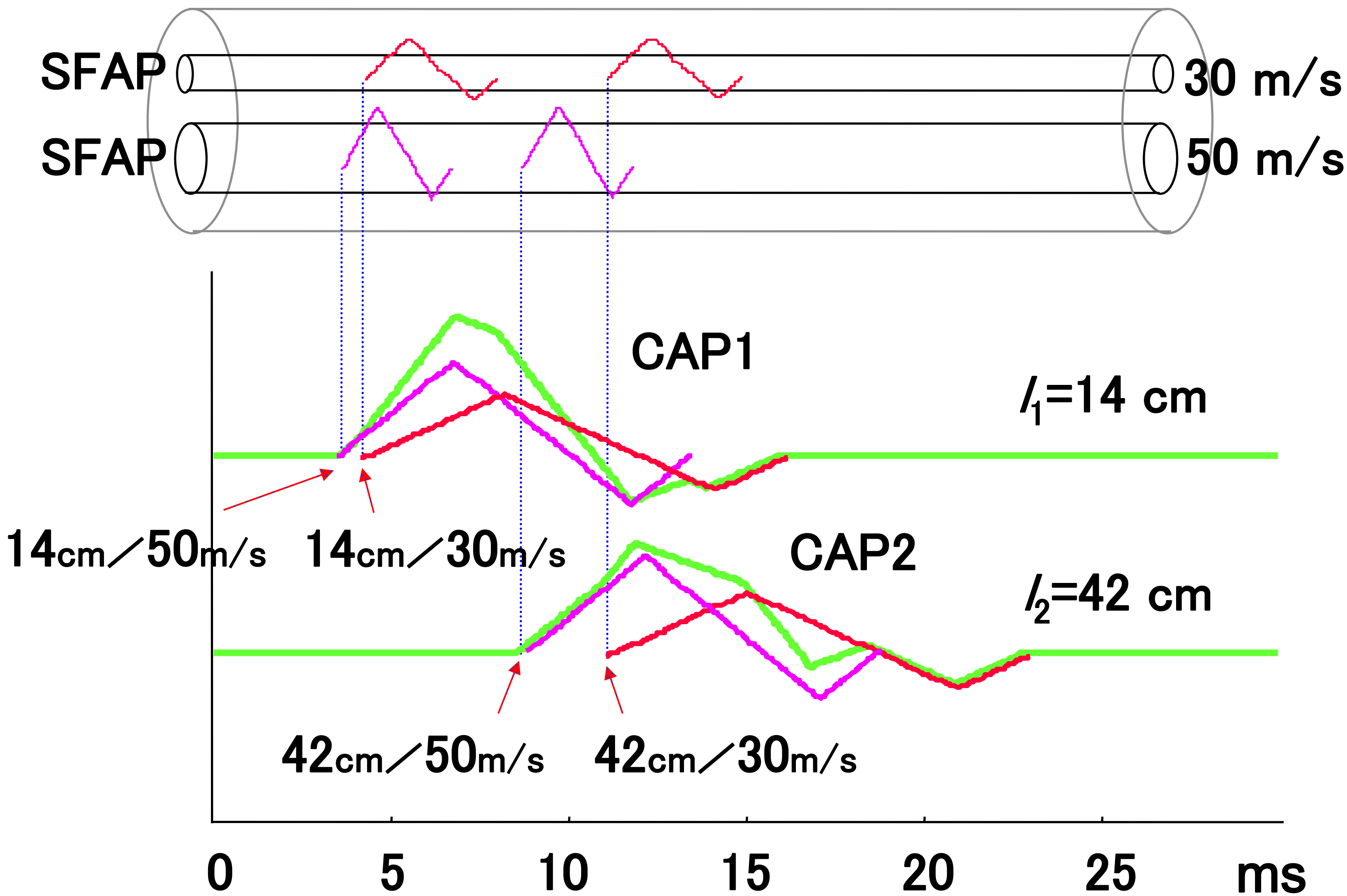




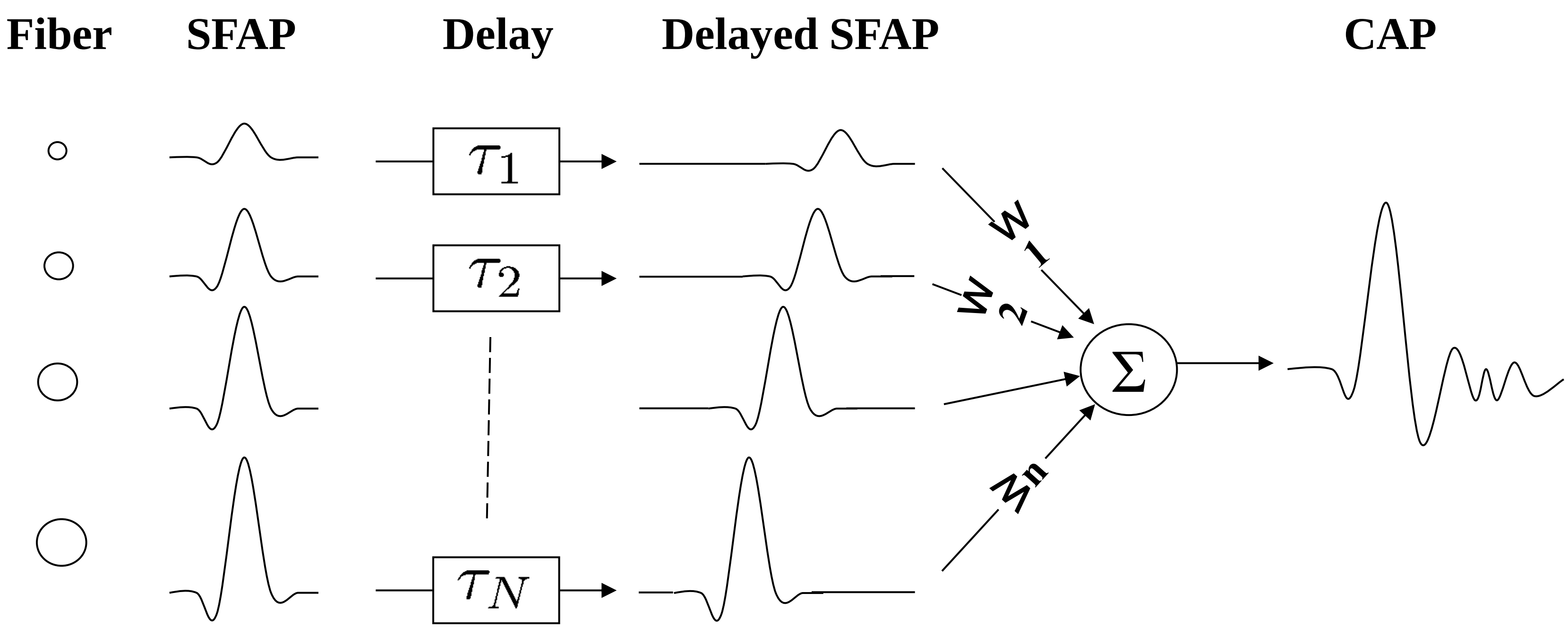
Collision Method



Single Fiber & Compound Action Potential



モデルの構築



$$y(t_i) = \sum_{j=1}^{N} A\ v_j^2\ w(v_j)\ s\left(t_i - \frac{l}{v_j}\right)$$

$y(t_i)$: CAP

$s(t_i)$: SFAP

$w(v_j)$: CVD

1. Linear superposition
2. Independent propagation
3. Simultaneous activation at the same position
4. Constant velocity proportional to the diameter
5. Same duration
6. Amplitude proportional to fiber cross section

Linear CAP model:

$$y(t) = \int_0^\infty A v^2 w(v) s\left(t - \frac{l}{v}\right) dv,$$

Formulation of the Problem

$$y(t) = \int_0^\infty A v^2 w(v) s(t - \frac{l}{v}) dv$$

$$\Downarrow \quad \tau \equiv l/v$$

$$y(t) = \int_0^\infty A \left(\frac{l}{\tau}\right)^2 w\left(\frac{l}{\tau}\right) s(t-\tau)\frac{1}{\tau^2} d\tau.$$

$$y(t) = \int_0^\infty p(\tau; w) s(t-\tau) d\tau \equiv [p(\cdot; w) * s](t)$$

$$p(\tau; w) \equiv \frac{Al^2}{\tau^4} w\left(\frac{l}{\tau}\right)$$

$$y_i(t) = [p_i(\cdot\,; w) * s](t) + n_i(t), \qquad i = 1, 2$$

$$H_1(\beta\omega)=H_2(\omega) \tag{4}$$

where $\beta=l_2/l_1>1$.

Taking into account (4), we write (2) in frequency domain, for two CAP's, $C_1(\omega)$ and $C_2(\omega)$, and after some manipulation, we obtain

$$H_1(\beta\omega)=\frac{C_2(\omega)}{C_1(\omega)}H_1(\omega) \tag{5}$$

In other words, (5) shows how the higher frequency components of $H_1(\omega)$ can be determined iteratively by the lower ones. Starting by an initial guess of $H_1(\omega/\beta^n) \simeq H_1(0) =$ cst > 0 (the integral of $h_1(\tau)$ is strictly positive), the following formula is derived

$$H_1(\omega)=\frac{C_2(\omega/\beta)}{C_1(\omega/\beta)}\frac{C_2(\omega/\beta^2)}{C_1(\omega/\beta^2)}\cdots\cdots\frac{C_2(\omega/\beta^n)}{C_1(\omega/\beta^n)}H_1(\omega/\beta^n) \tag{6}$$

Formulation of the Problem

$$y(t) = \int_0^\infty Av^2 w(v) s(t - \frac{l}{v}) dv$$

$$\Downarrow \quad \tau \equiv l/v$$

$$y(t) = \int_0^\infty A \left(\frac{l}{\tau}\right)^2 w\left(\frac{l}{\tau}\right) s(t-\tau) \frac{1}{\tau^2} d\tau.$$

$$y(t) = \int_0^\infty p(\tau; w) s(t-\tau) d\tau \equiv [p(\cdot; w) * s](t)$$

$$p(\tau; w) \equiv \frac{Al^2}{\tau^4} w\left(\frac{l}{\tau}\right)$$

$$y_i(t) = [p_i(\cdot\, ; w) * s](t) + n_i(t), \qquad i = 1, 2$$

BLIND DECONVOLUTION

CAPs

$$\begin{aligned} y_1(t) &= (p_1 * s)(t) + n_1(t) \\ y_2(t) &= (p_2 * s)(t) + n_2(t) \end{aligned}$$

Latency & DCV:

$$p_i(\tau) = \frac{A l_i^2}{\tau^4} w\left(\frac{l_i}{\tau}\right) = \frac{A}{l_i^2}\ v^4\ w(v), \qquad i = 1, 2$$

Blind Deconvlution Problem:

$$J(s, w; l_1, l_2) \equiv \|y_1 - p_1(\cdot\,; w) * s\|^2 + \|y_2 - p_2(\cdot\,; w) * s\|^2 \to \text{minimize}$$

Two Stage Algorithm:

$$[\tilde{s}(t;l_1,l_2),\, \tilde{w}(v;l_1,l_2)] \quad = \quad \operatorname*{argmin}_{s,w} J(s,w;l_1,l_2)$$

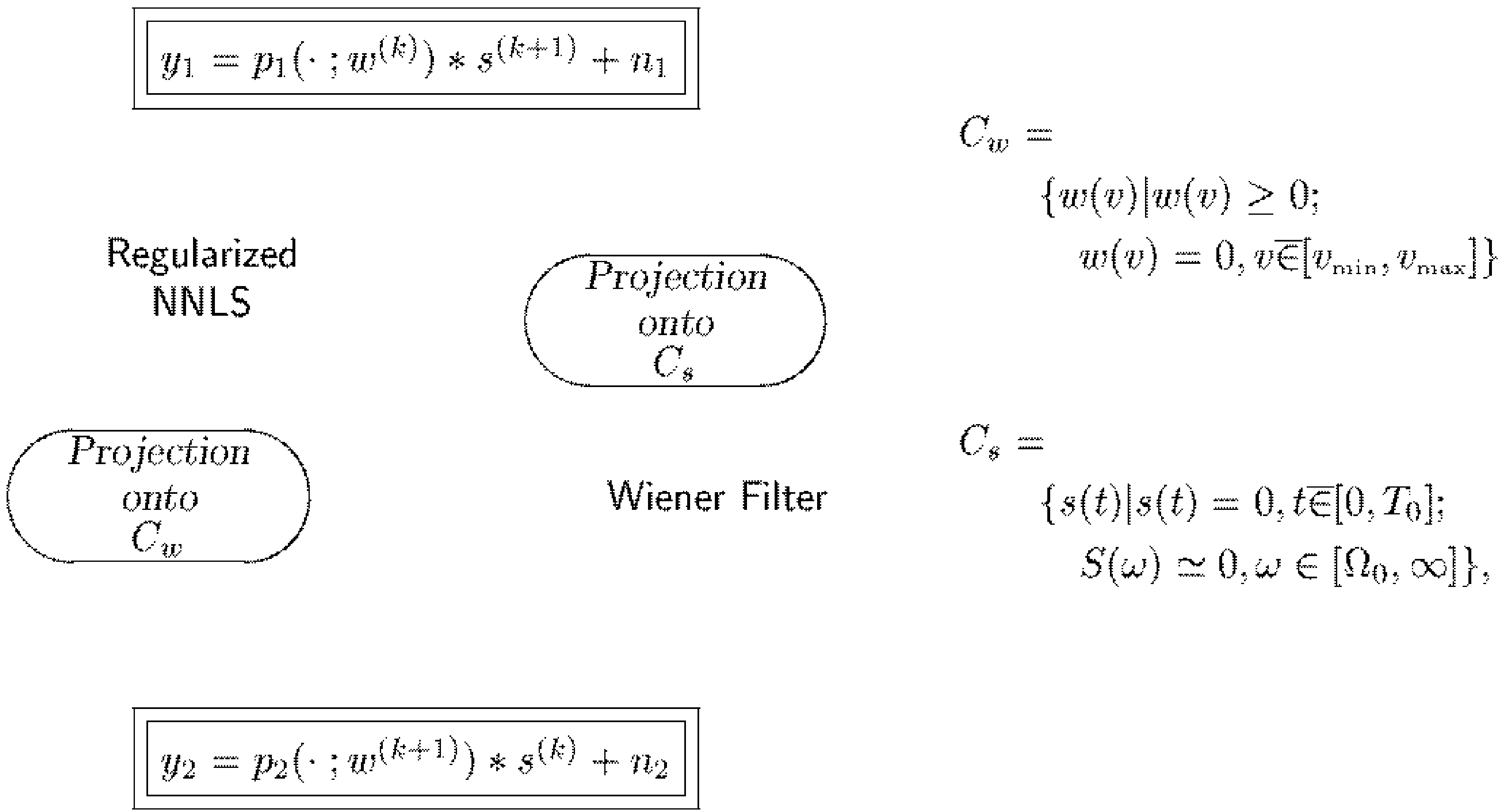
$$\cdots \quad \text{GPOCS – RNNLS \& WF}$$

$$\left[\hat{l}_1,\hat{l}_2\right] \quad = \quad \operatorname*{argmin}_{l_1,l_2} J(\tilde{s}(\cdot;l_1,l_2),\tilde{w}(\cdot;l_1,l_2);l_1,l_2)$$

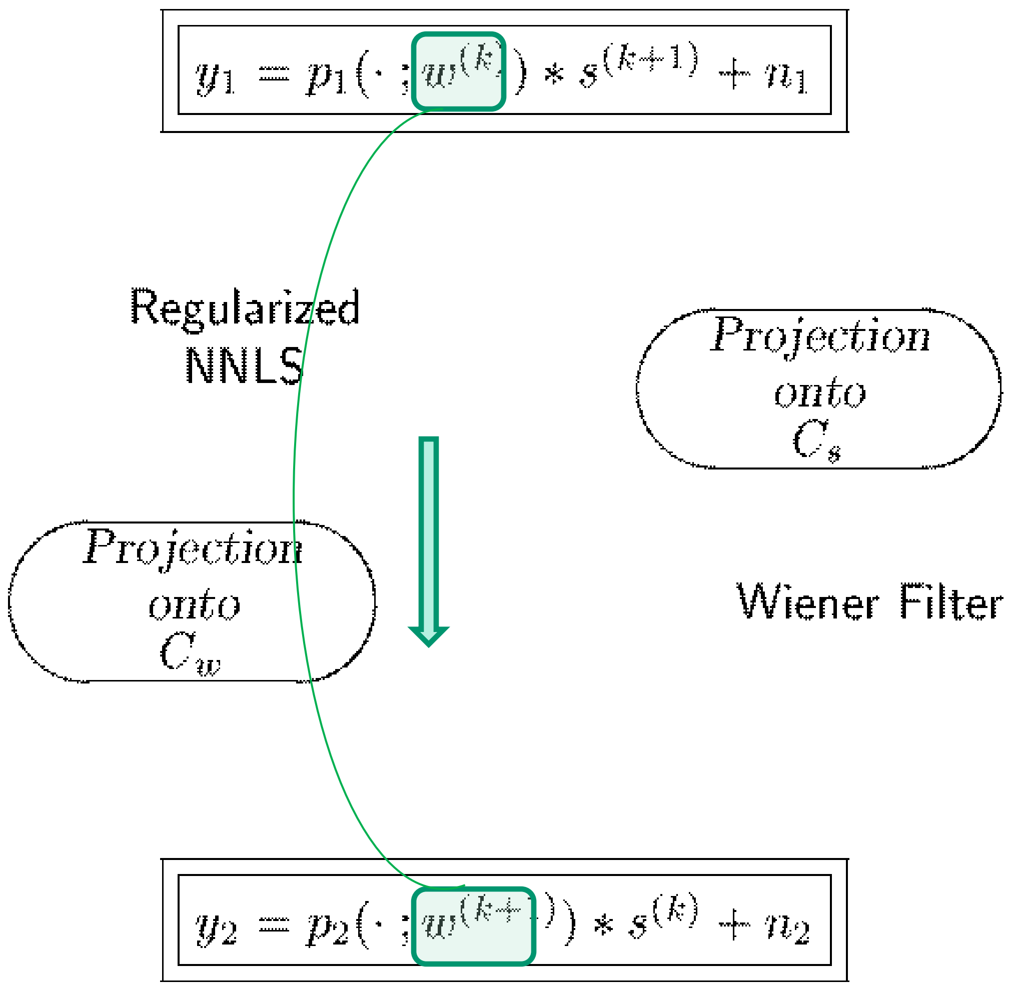
$$\cdots \quad \text{Fibonacci Search}$$

$$\hat{s}(t) \quad = \quad \tilde{s}(t\ ;\hat{l_1},\hat{l_2})$$

$$\hat{w}(v) \quad = \quad \tilde{w}(v\ ;\hat{l_1},\hat{l_2})$$

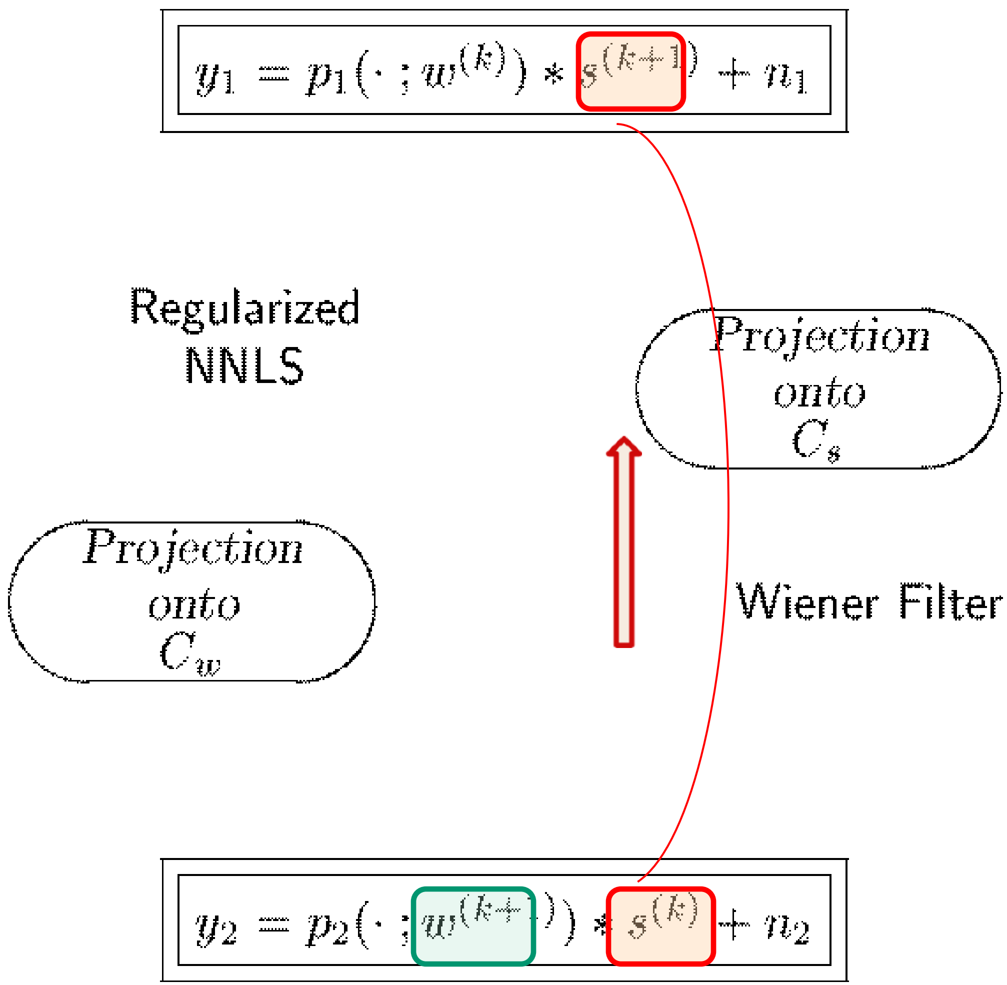


$y_1 = p_1(\cdot\,; w^{(k)}) * s^{(k+1)} + n_1$
$C_w =$
$\{w(v) | w(v) \geq 0;$
$w(v) = 0, v \overline{\in} [v_{\min}, v_{\max}]\},$
Regularized
NNLS
Projection onto C_s
Projection onto C_w
Wiener Filter
$C_s =$
$\{s(t) | s(t) = 0, t \overline{\in} [0, T_0];$
$S(\omega) \simeq 0, \omega \in [\Omega_0, \infty]\},$
$y_2 = p_2(\cdot\,; w^{(k+1)}) * s^{(k)} + n_2$



$$C_w = \{w(v) | w(v) \geq 0; \; w(v) = 0, v \overline{\in} [v_{\min}, v_{\max}]\},$$

$$C_s = \{s(t) | s(t) = 0, t \overline{\in} [0, T_0]; \; S(\omega) \simeq 0, \omega \in [\Omega_0, \infty]\},$$



$$C_w = \{w(v) \mid w(v) \geq 0;\ w(v) = 0, v \overline{\in} [v_{\min}, v_{\max}]\},$$

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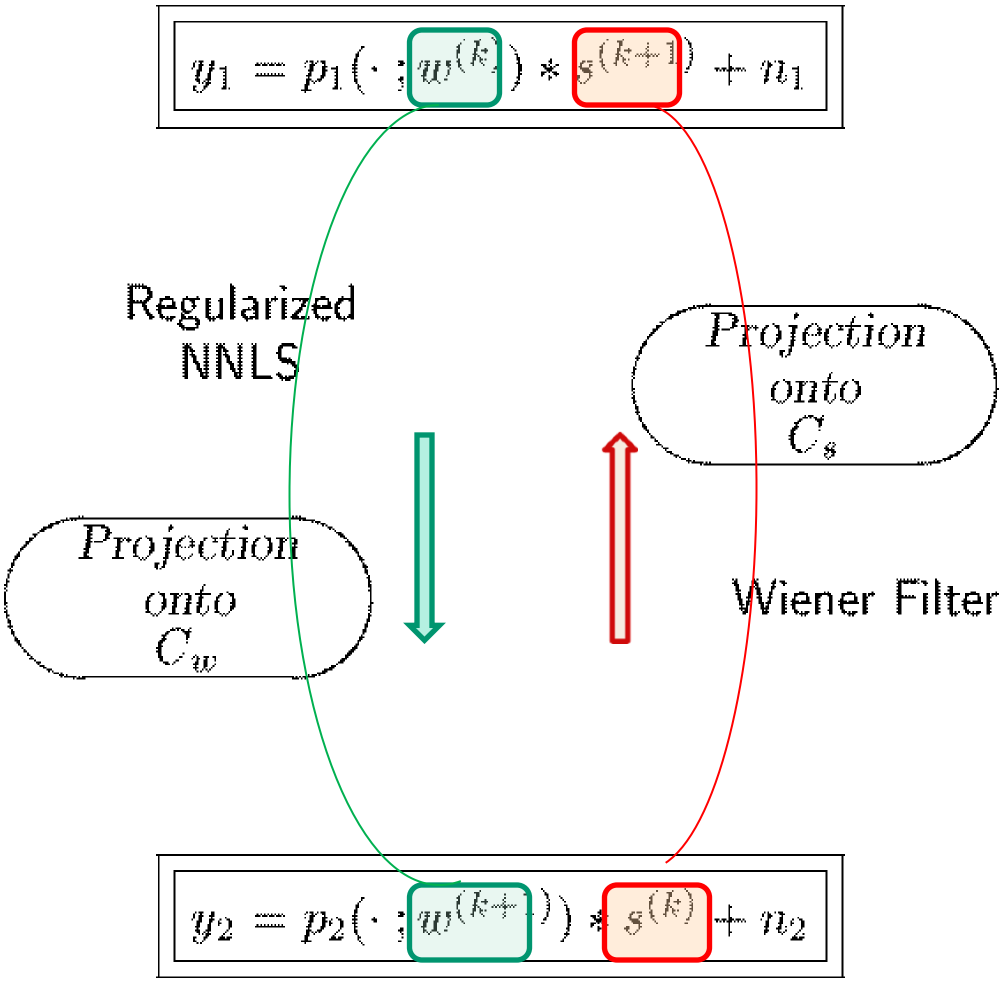


$$C_w = \{w(v) | w(v) \geq 0; \; w(v) = 0, v \overline{\in} [v_{\min}, v_{\max}]\},$$

$$C_s = \{s(t) | s(t) = 0, t \overline{\in} [0, T_0]; \; S(\omega) \simeq 0, \omega \in [\Omega_0, \infty]\},$$

Regularized NonNegative Least Square (RNNLS)

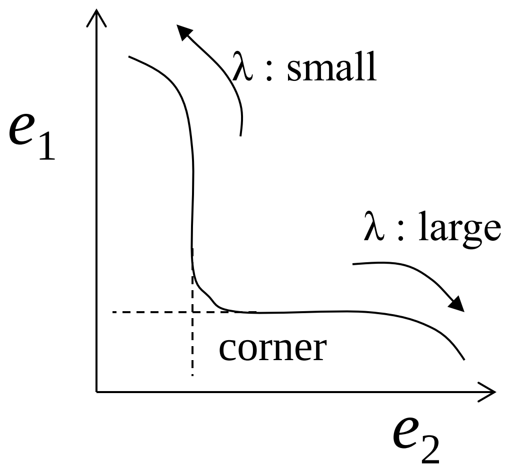
$$\mathbf{y} = \mathbf{A}\mathbf{w}$$

$$e = \underbrace{\left\| \mathbf{y} - \mathbf{A}\mathbf{w} \right\|^2}_{e_1} + \lambda \underbrace{\left\| \mathbf{C}\mathbf{w} \right\|^2}_{e_2} \quad \rightarrow \quad \min_{\mathbf{w} \geq 0}$$

L-curve

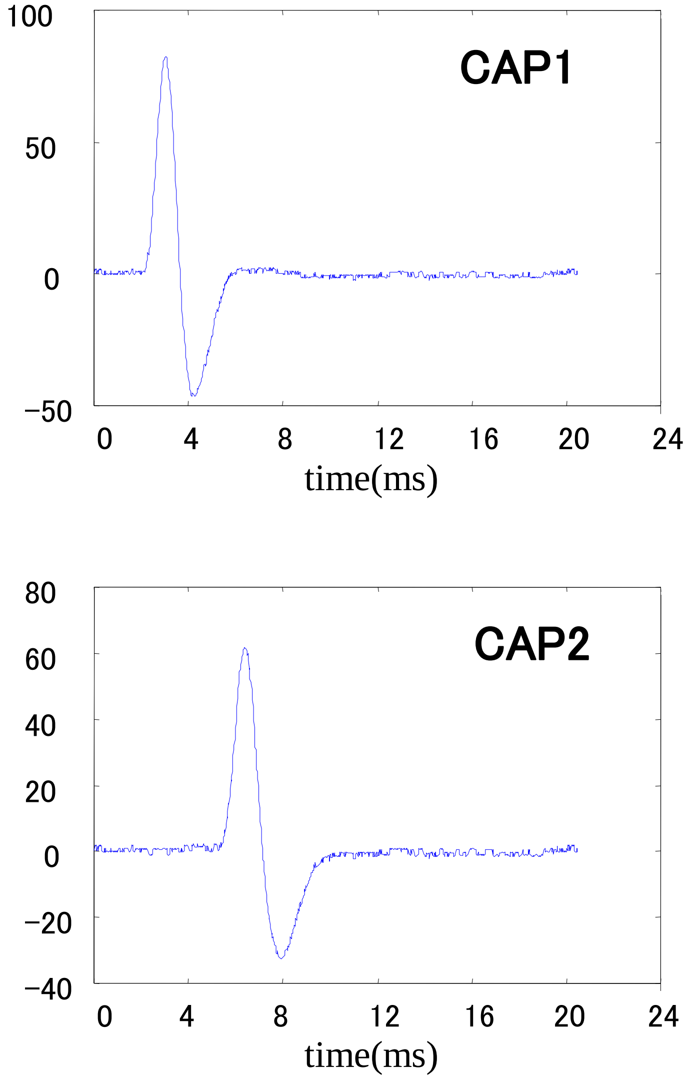
λ: regularization param.

C : diffrential op.

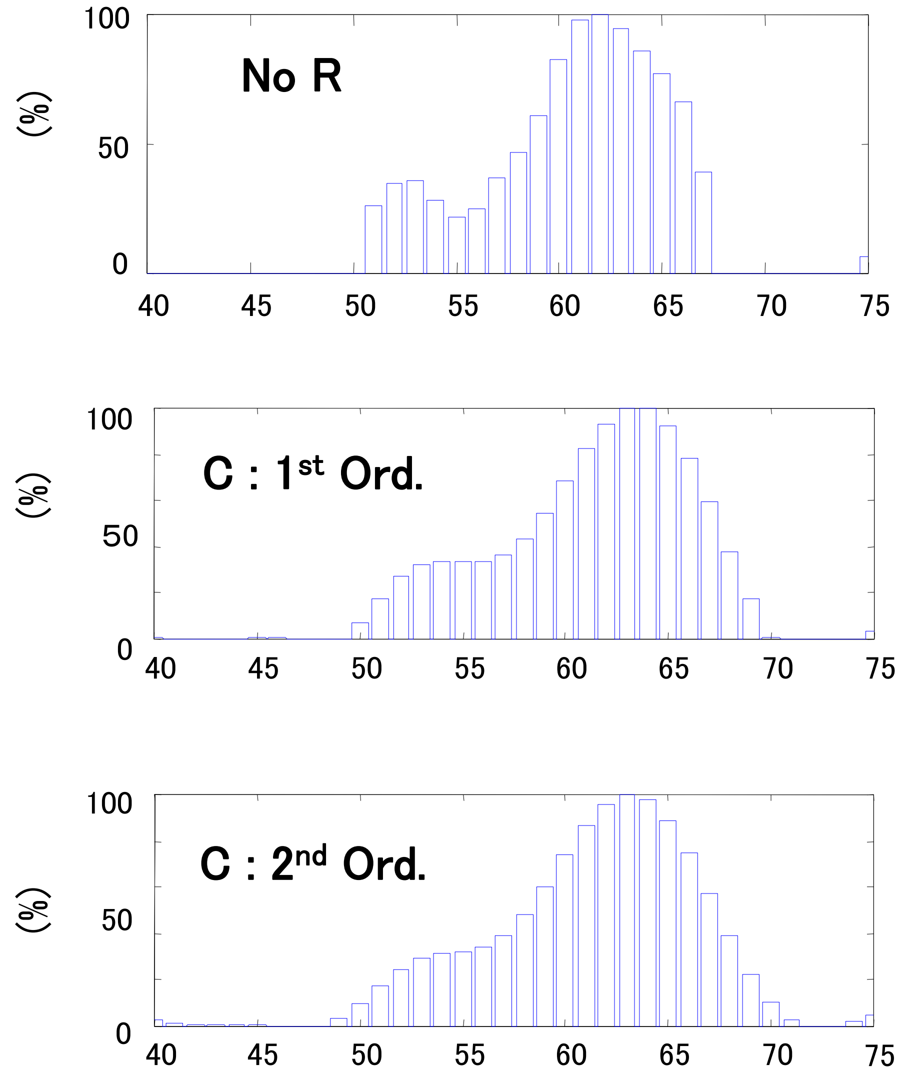


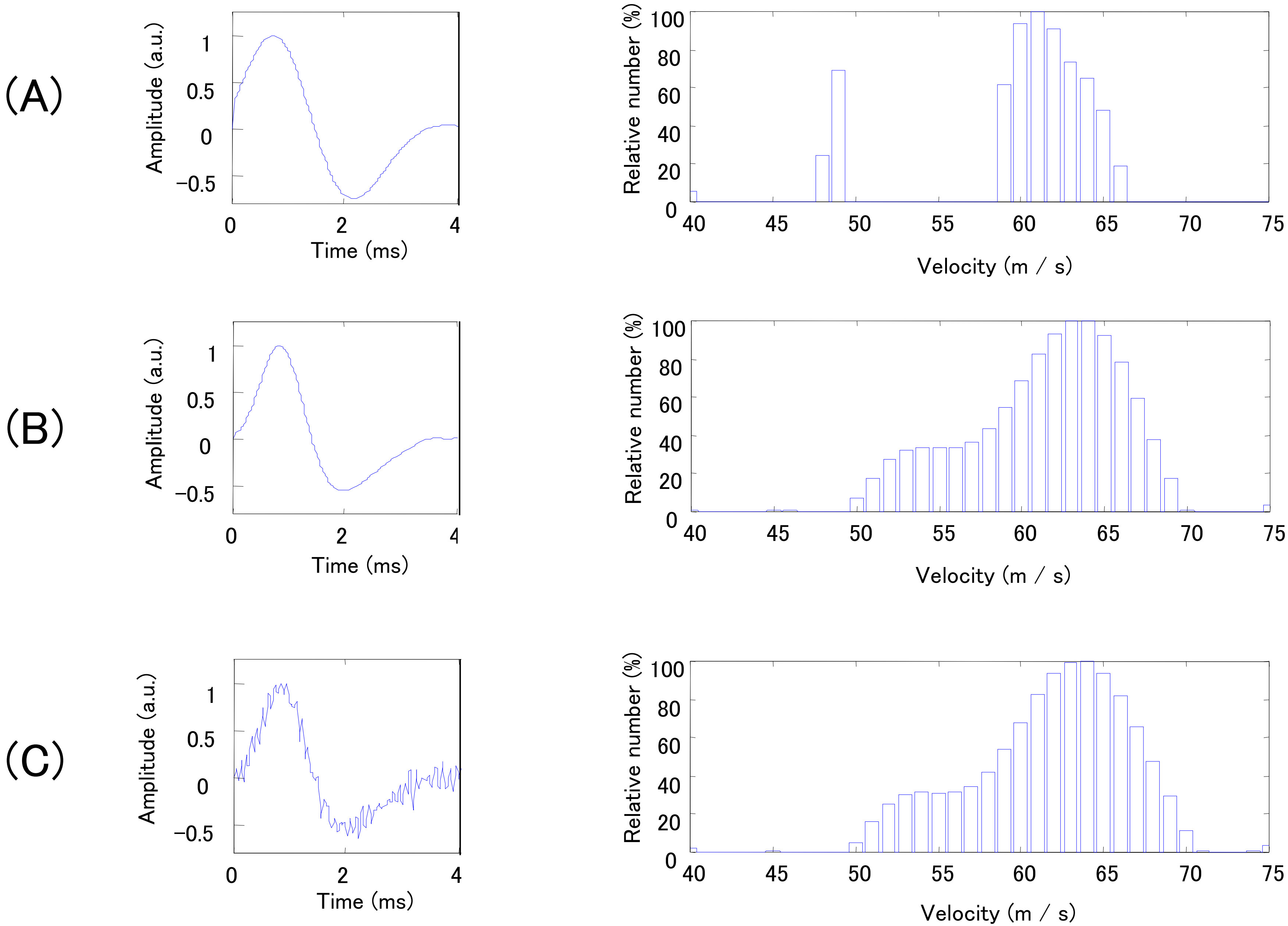
Effect of Differential Operators

CAP

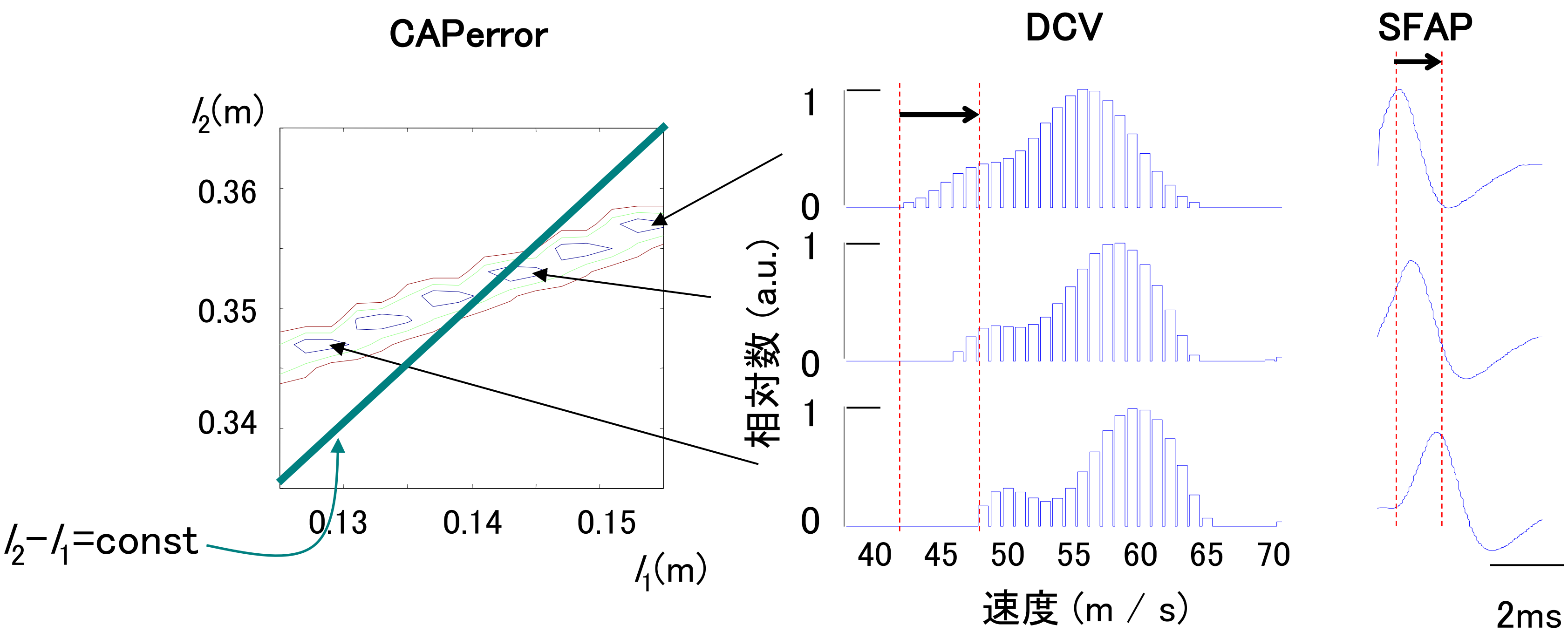


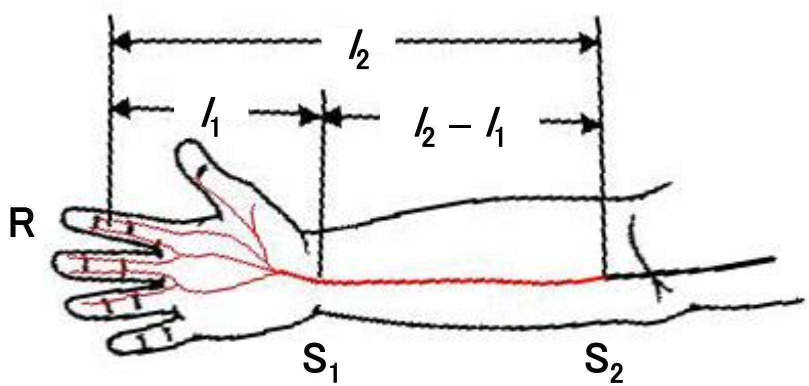
DCV




(A)
(B)
(C)
Amplitude (a.u.)
1
0.5
0
−0.5
0
2
4
Time (ms)
Relative number (%)
100
80
60
40
20
0
40
45
50
55
60
65
70
75
Velocity (m / s)

Optimal search of conduction length l_1, l_2





① l_2-l_1 ← measured value

② search for l_1

Two Stage Algorithm:

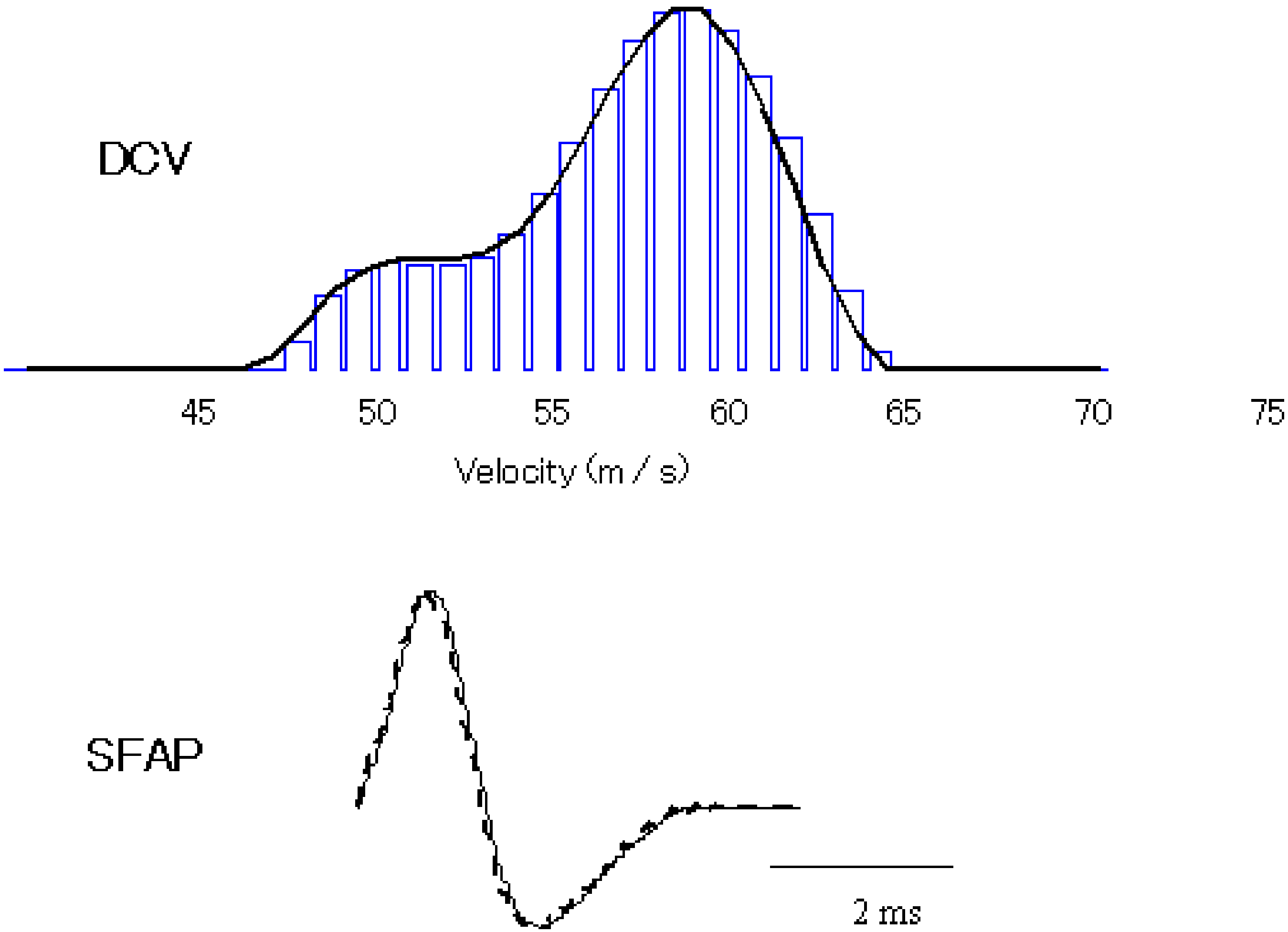
$$[\tilde{s}(t; l_1, l_2), \tilde{w}(v; l_1, l_2)] = \underset{s,w}{\text{argmin}}\, J(s, w; l_1, l_2)$$

$\cdots$ GPOCS – RNNLS & WF

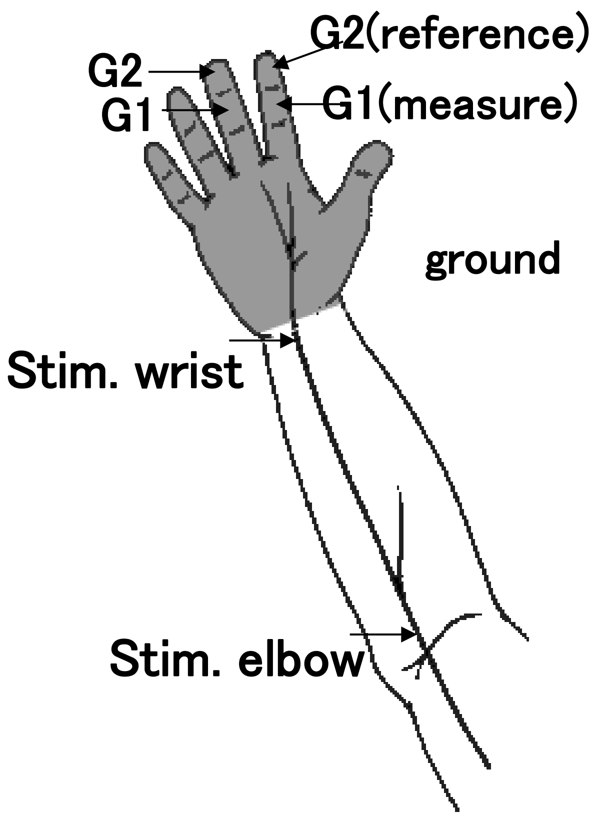
$$\left[\hat{l}_1, \hat{l}_2\right] = \underset{l_1, l_2}{\text{argmin}}\, J(\tilde{s}(\cdot; l_1, l_2), \tilde{w}(\cdot; l_1, l_2); l_1, l_2)$$

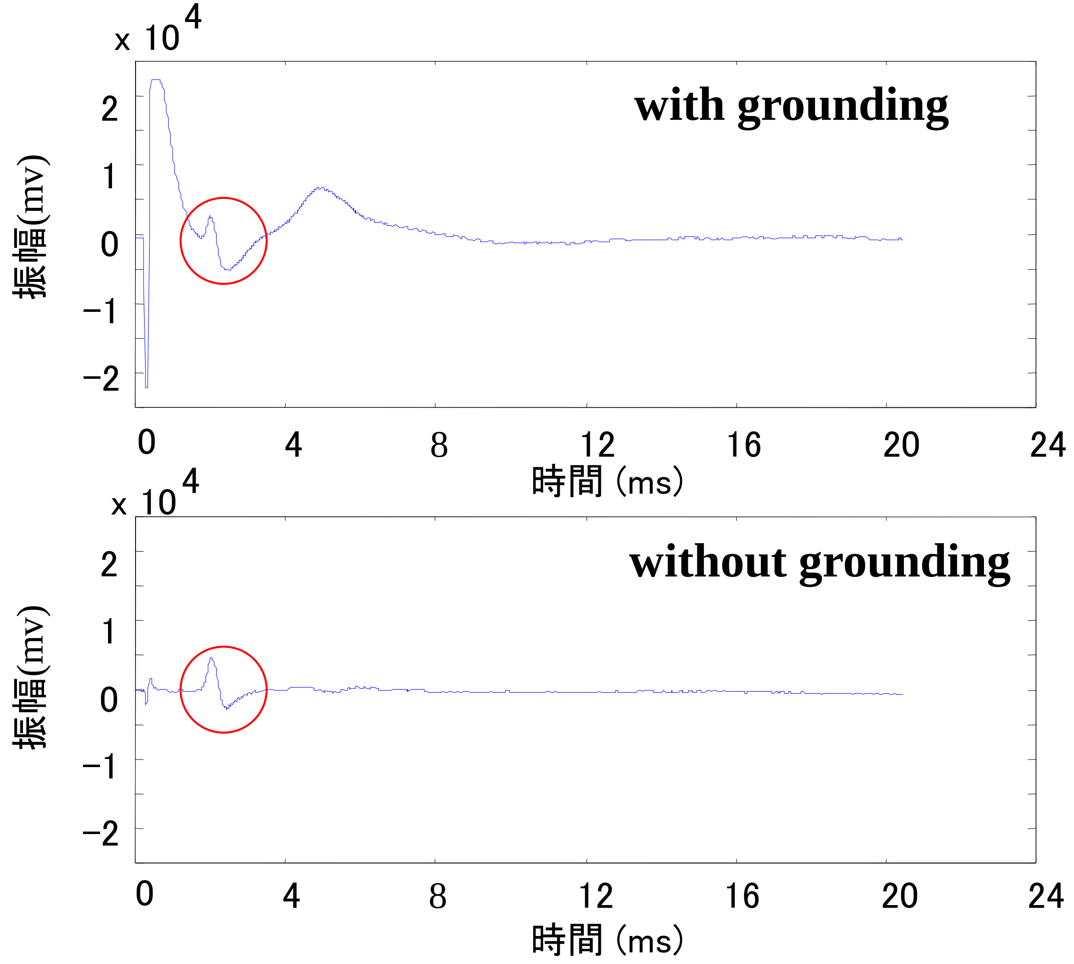
$\cdots$ Fibonacci Search

$$\hat{s}(t) = \tilde{s}(t\ ; \hat{l_1}, \hat{l_2})$$
$$\hat{w}(v) = \tilde{w}(v\ ; \hat{l_1}, \hat{l_2})$$


DCV
45
50
55
60
65
70
75
Velocity (m / s)
SFAP
2 ms

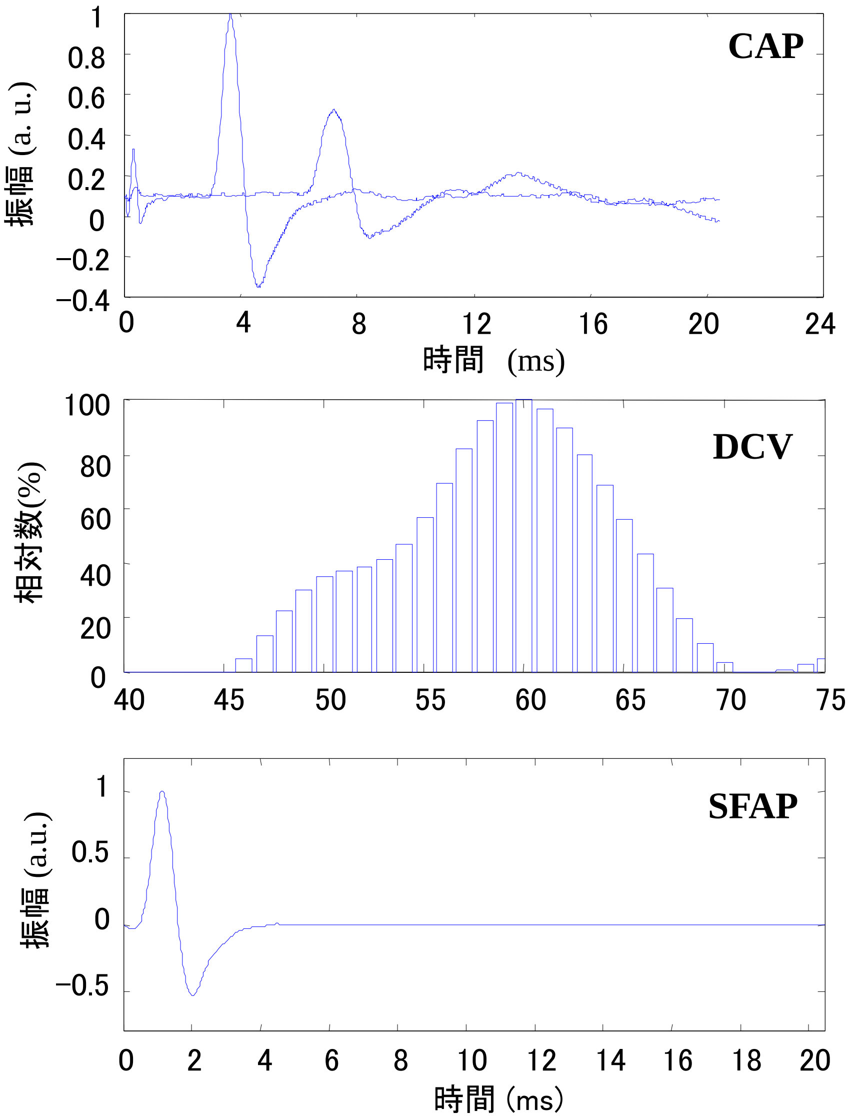
CAP計測における手のひらアースの効果



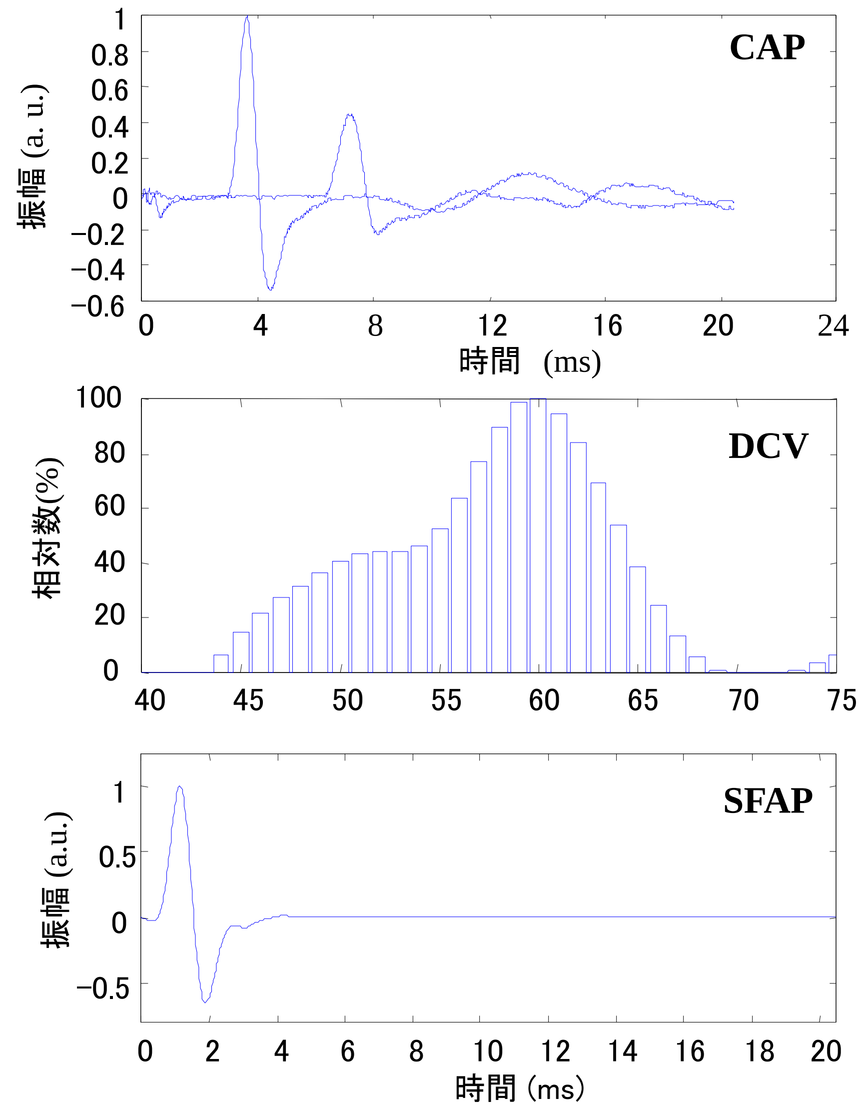


Sound subject
(#1)

corrcoeff.=0.969



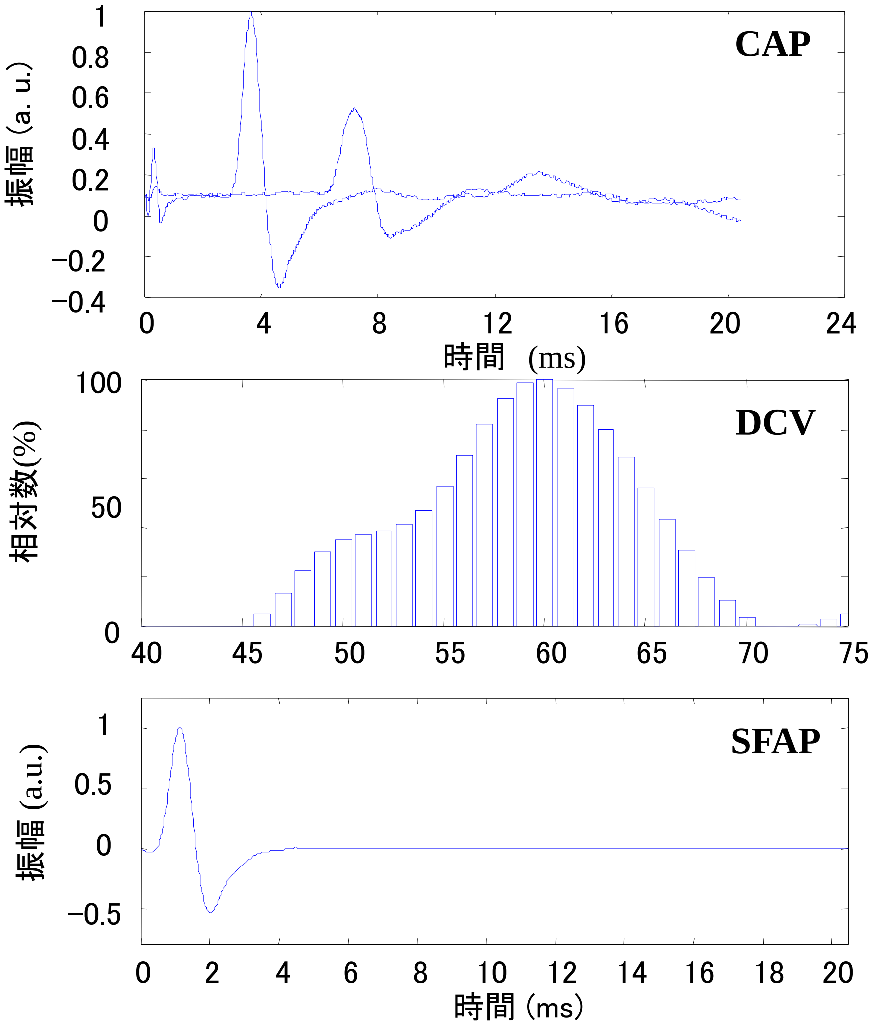
index finger



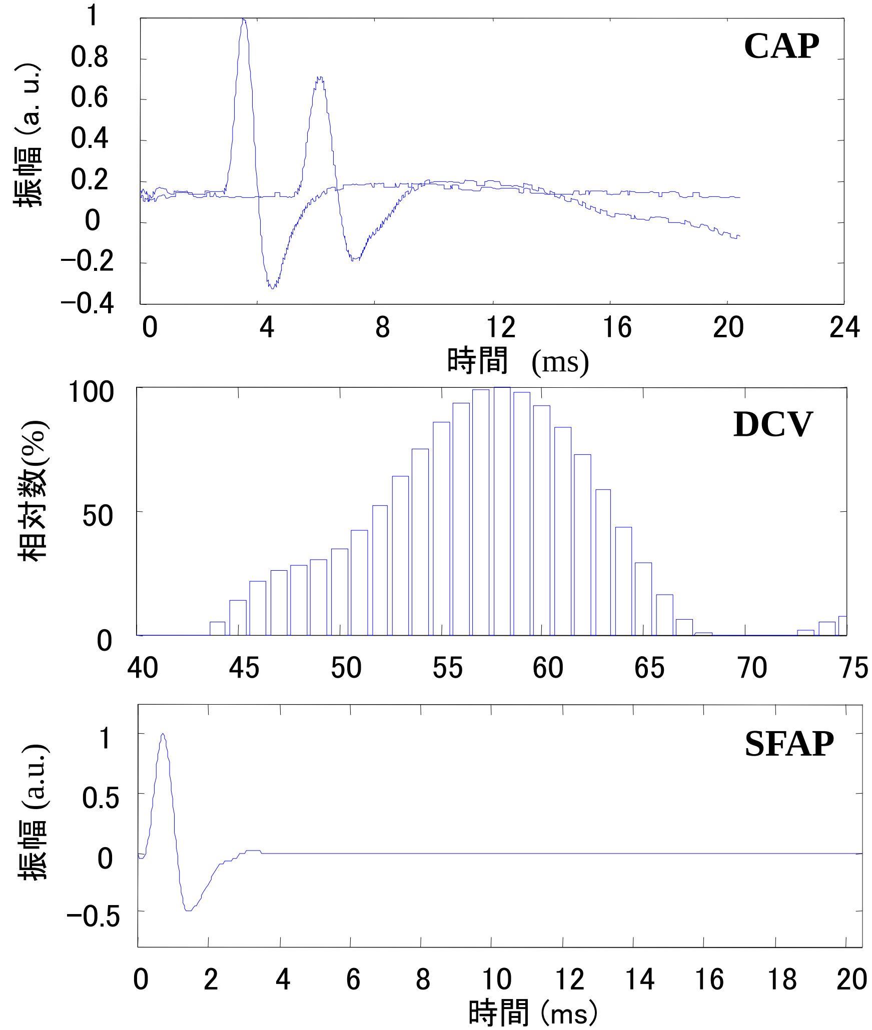
middle finger

Sound su bject
(#2)

corrcoef=0.909



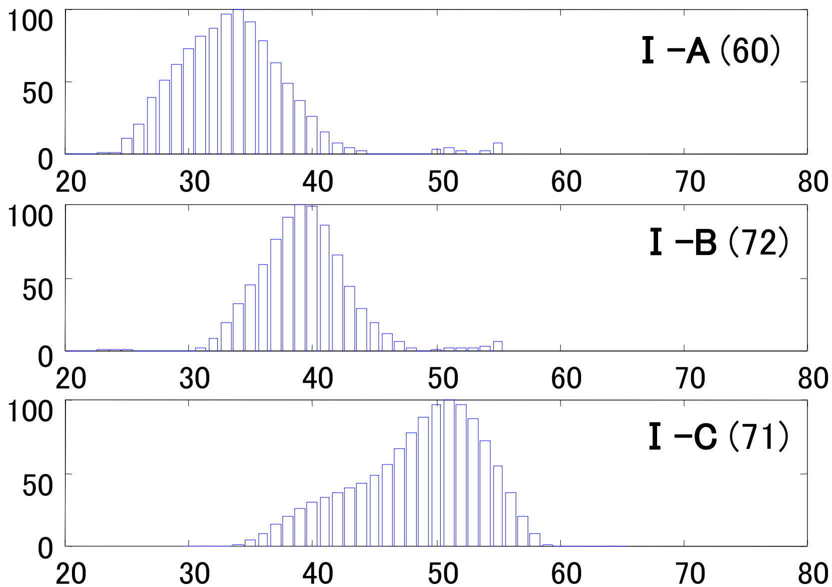
(A) 2000. Mar



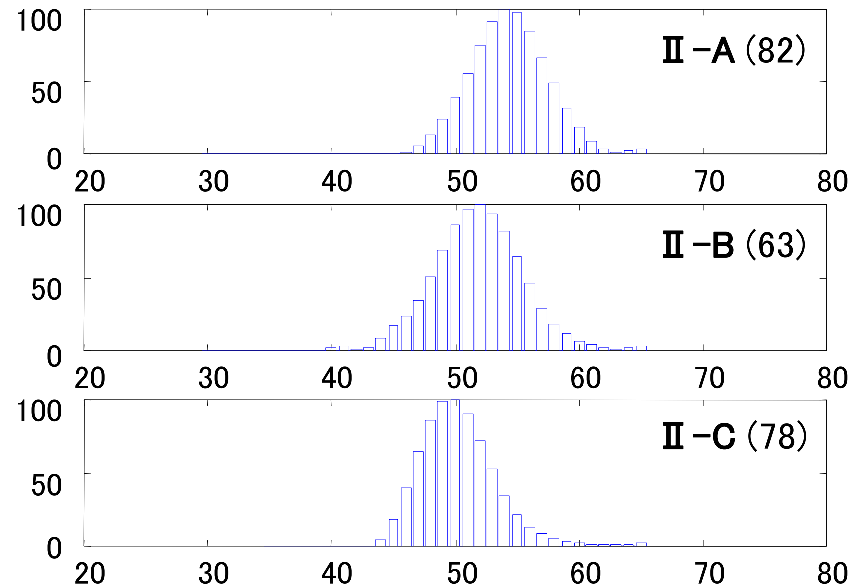
(B) 2000. Oct

CDV of diabetes patients

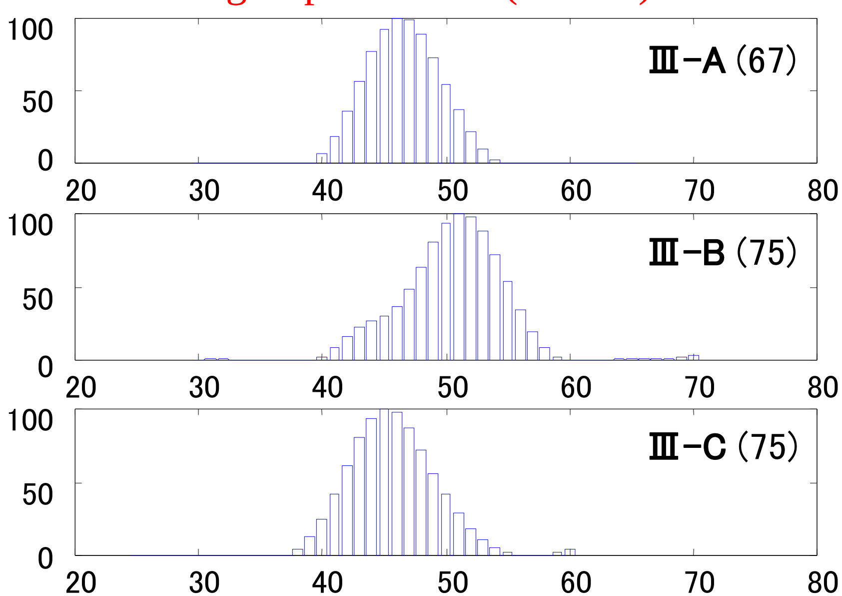
group Ⅰ : diabetes with neural disorder



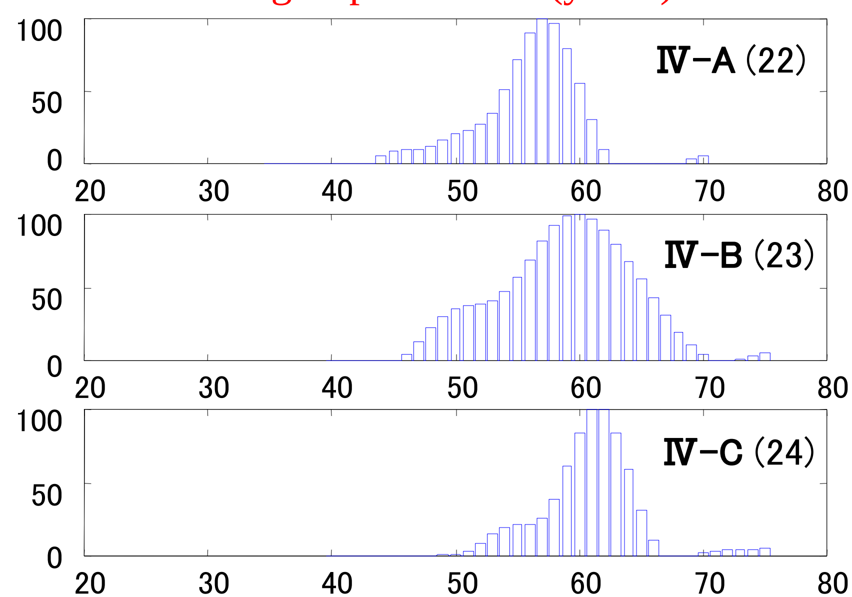
group Ⅱ : diabetes without neural disorder



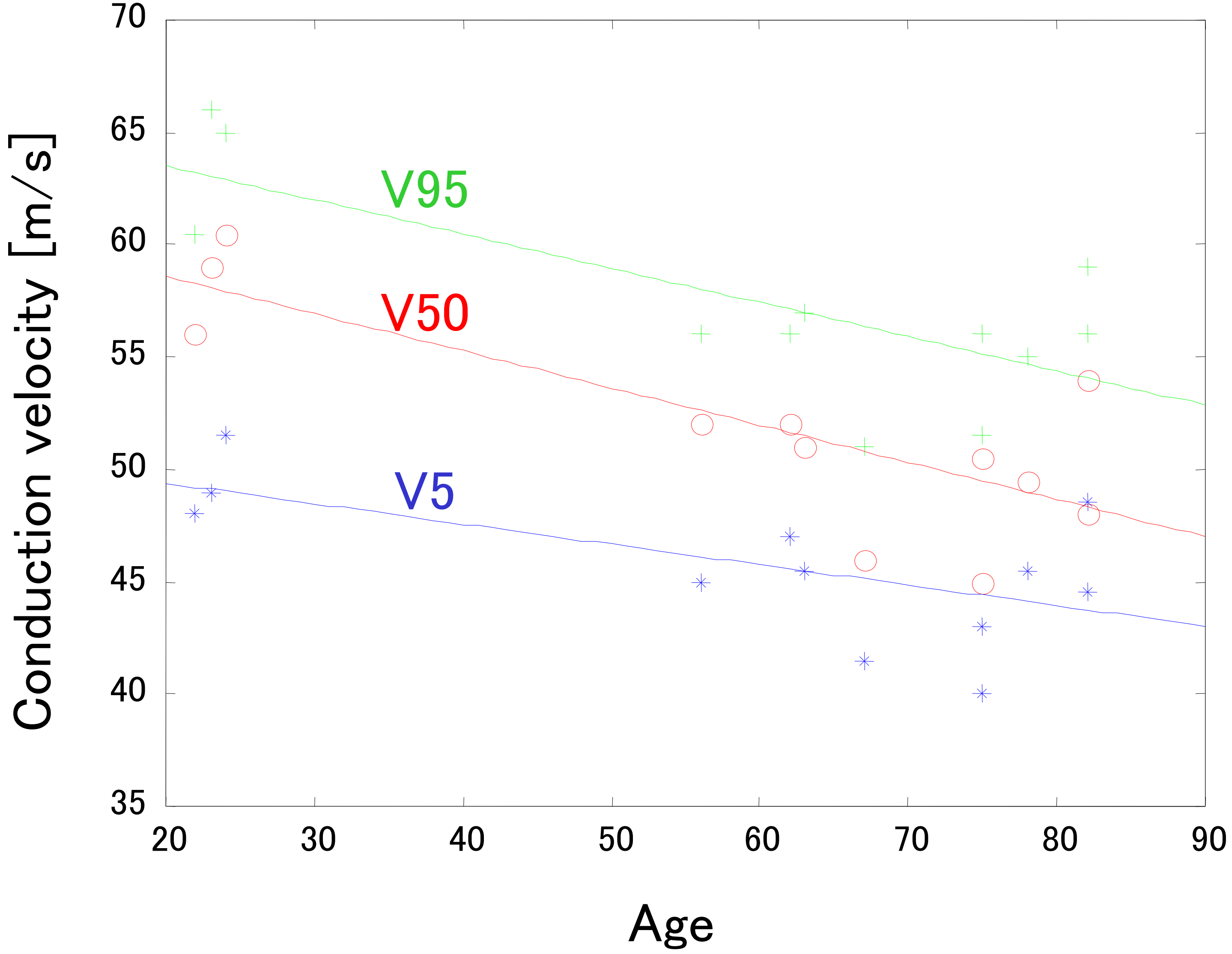
group Ⅲ : sound(control)



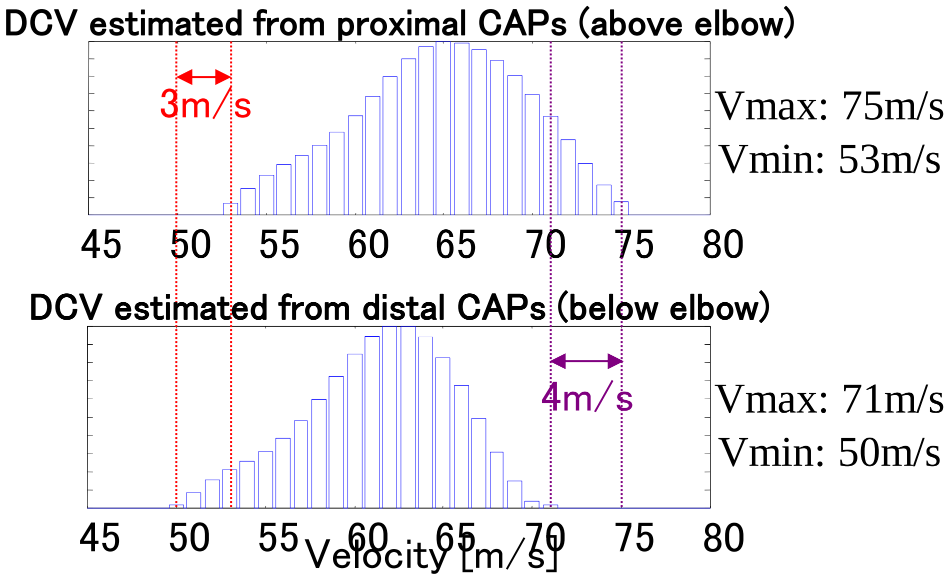
group Ⅳ : sound(youth)



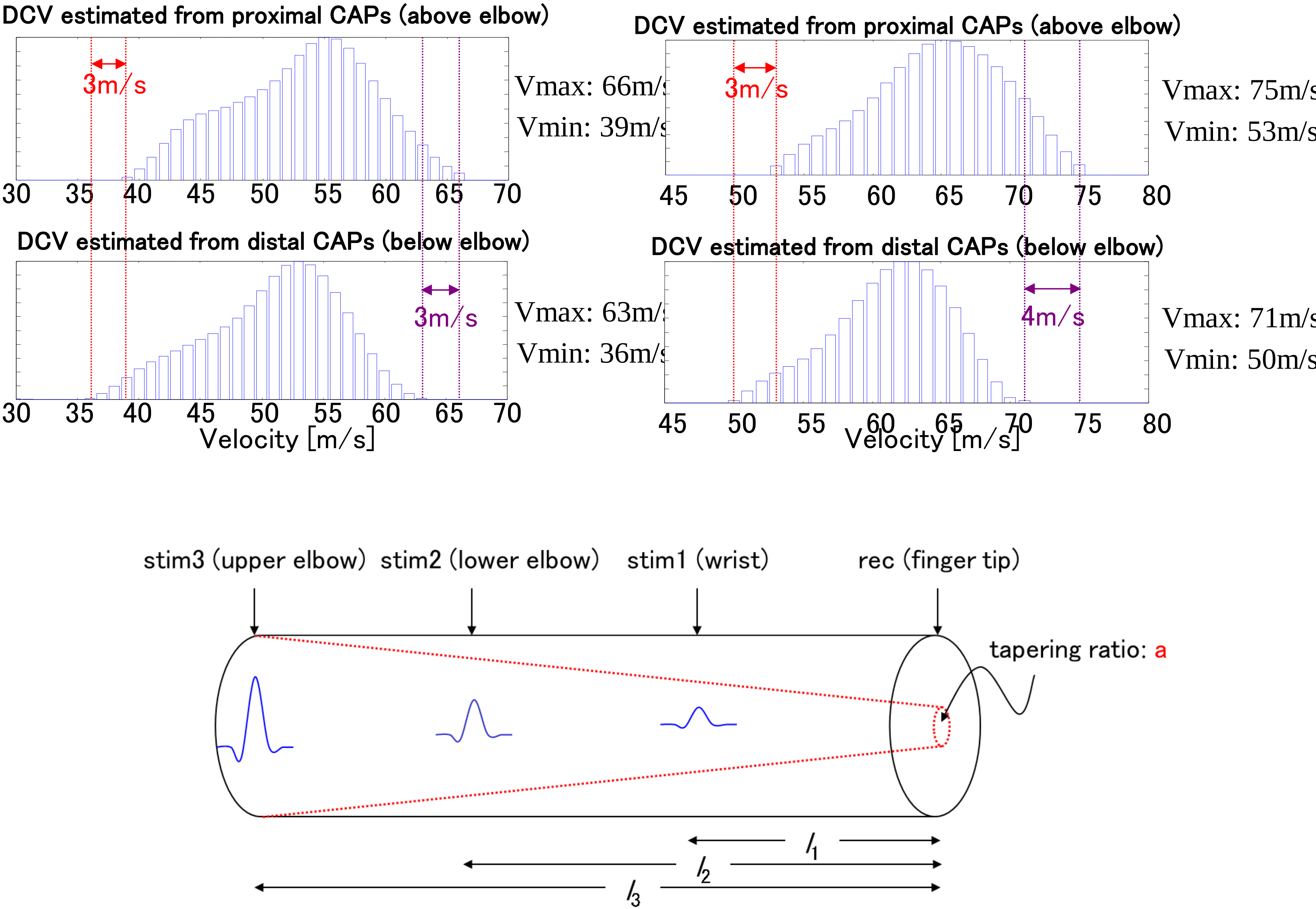
Aging of CVD

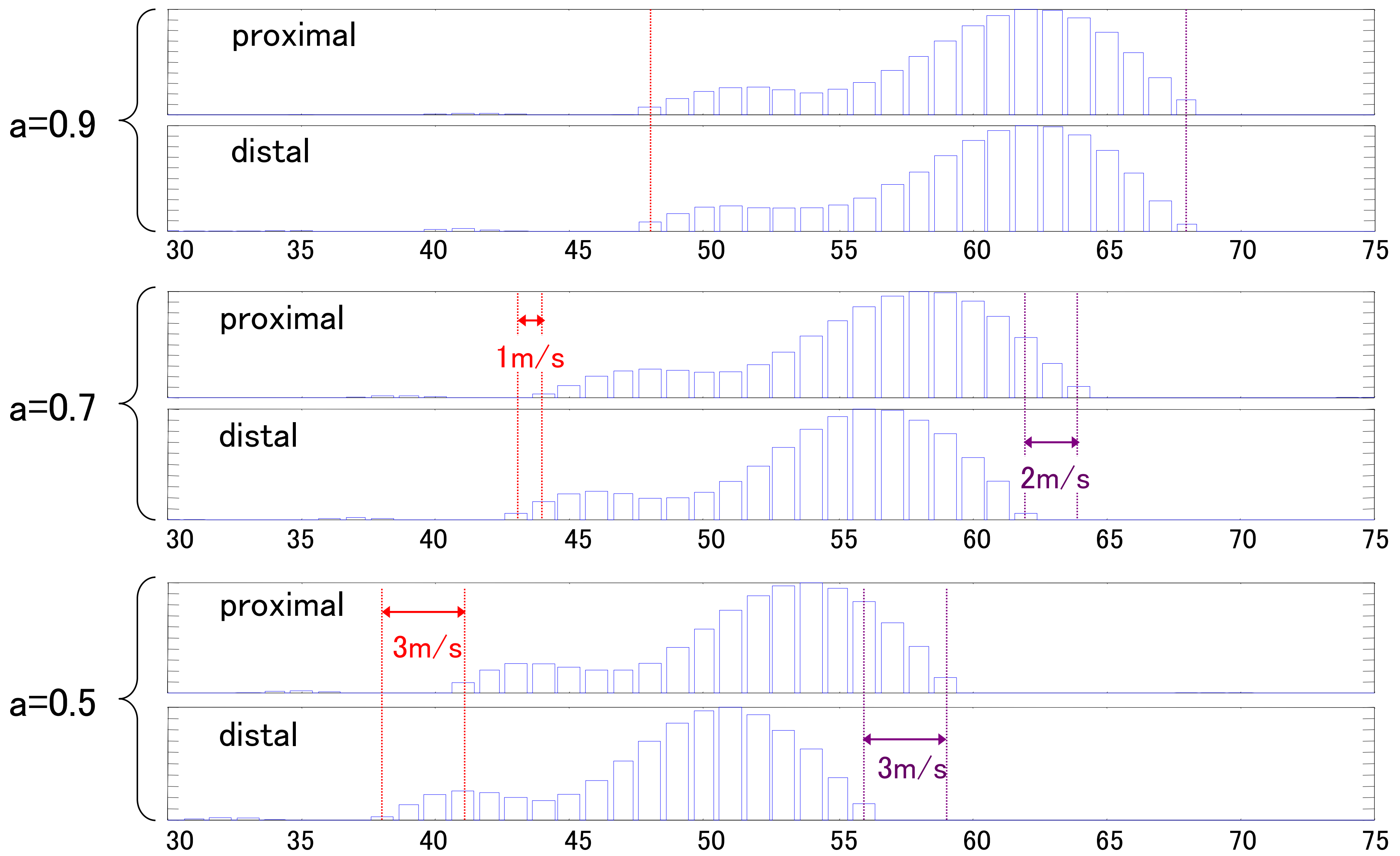


Conduction velocity [m/s]
70
65
60
55
50
45
40
35
20
30
40
50
60
70
80
90
Age
V95
V50
V5



DCV estimated from proximal CAPs (above elbow)
3m/s
Vmax: 75m/s
Vmin: 53m/s
45
50
55
60
65
70
75
80
DCV estimated from distal CAPs (below elbow)
4m/s
Vmax: 71m/s
Vmin: 50m/s
45
50
55
60
65
70
75
80
Velocity [m/s]


DCV estimated from proximal CAPs (above elbow)
3m/s
Vmax: 66m/s
Vmin: 39m/s
DCV estimated from distal CAPs (below elbow)
3m/s
Vmax: 63m/s
Vmin: 36m/s
Velocity [m/s]
DCV estimated from proximal CAPs (above elbow)
3m/s
Vmax: 75m/s
Vmin: 53m/s
DCV estimated from distal CAPs (below elbow)
4m/s
Vmax: 71m/s
Vmin: 50m/s
Velocity [m/s]
stim3 (upper elbow)
stim2 (lower elbow)
stim1 (wrist)
rec (finger tip)
tapering ratio: a
l_1
l_2
l_3


a=0.9
proximal
distal
a=0.7
proximal
1m/s
distal
2m/s
a=0.5
proximal
3m/s
distal
3m/s
30
35
40
45
50
55
60
65
70
75

Inversion of the Causality

Thank your very much