Tomography Methods for Tangential Imaging

National Institute for Fusion Science
S. Ohdachi

Outline of my talk

1. Merit of the tangential imaging system for fluctuation measurements. (Esp. Island study)
2. Need for the reconstruction of the radiation profile at the poloidal cross section.

3. Tomographic reconstruction methods developed in 2D tomography. Assumptions in the reconstruction
   – equivalent line of sights
   toroidal symmetry / helical symmetry / magnetic field line

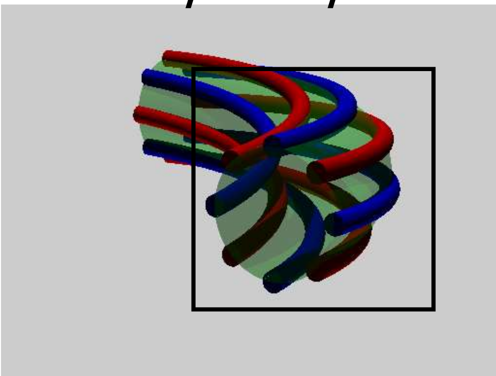
4. SVD method for separation of the fluctuating components
   SVD → Reconstruction / Reconstruction → SVD
5. Sample of the reconstruction.
6. Summary

Thanks to the LHD experiment group, TEXTOR team,
IEA agreement between JAPAN and TEXTOR.

Merit of the tangentially viewing camera

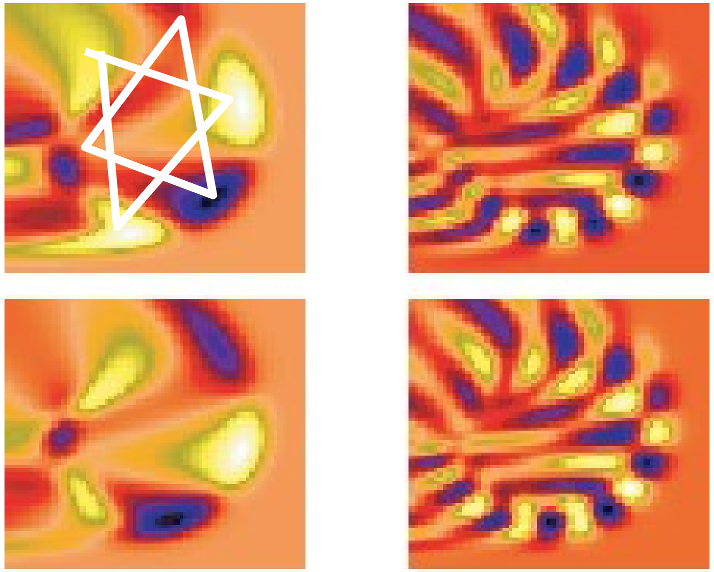
m/n=3/2

$k_{\perp} >> k_{//}$



Tangentially viewing
on simple torus

m/n=3/2　　m/n=10/5



- Poloidal mode number can be distinguished from the raw data easily without complicated reconstruction.
- When the perturbations are localized on magnetic field lines, tangentially viewing measurements give a good contrast even for high mode numbers.

History of tangential imaging (SX)

From X-ray radiation, we can study core plasma.

(Edge plasma for Bolometer, Visible lights measurements)

- 1980s
  - Nagoya Univ. (CLEO, MCP, S. Takamura et. al., Nucl. Fusion Vol. 23 (1983) 1485)
  - PPPL (PBX-M, P20+MCP, R. J. Fonck et. al., RSI Vol. 59 (1988)1831)

Hard X-ray(super thermal elc.)

- 1990s
  - PPPL (PBX-M, SXII, S. von Goeler et. al., RSI Vol. 65 (1994)1621)

Estimation of equilibrium

  - NIFS (LHD, SX CCD, Y. Liang et. al., RSI, Vol.72 (2001)717)
  - PPPL + IPP Juelich + NIFS

Fluctuation study

    (Concept, S. von Goeler, RSI Vol. 61(1990) 3055)

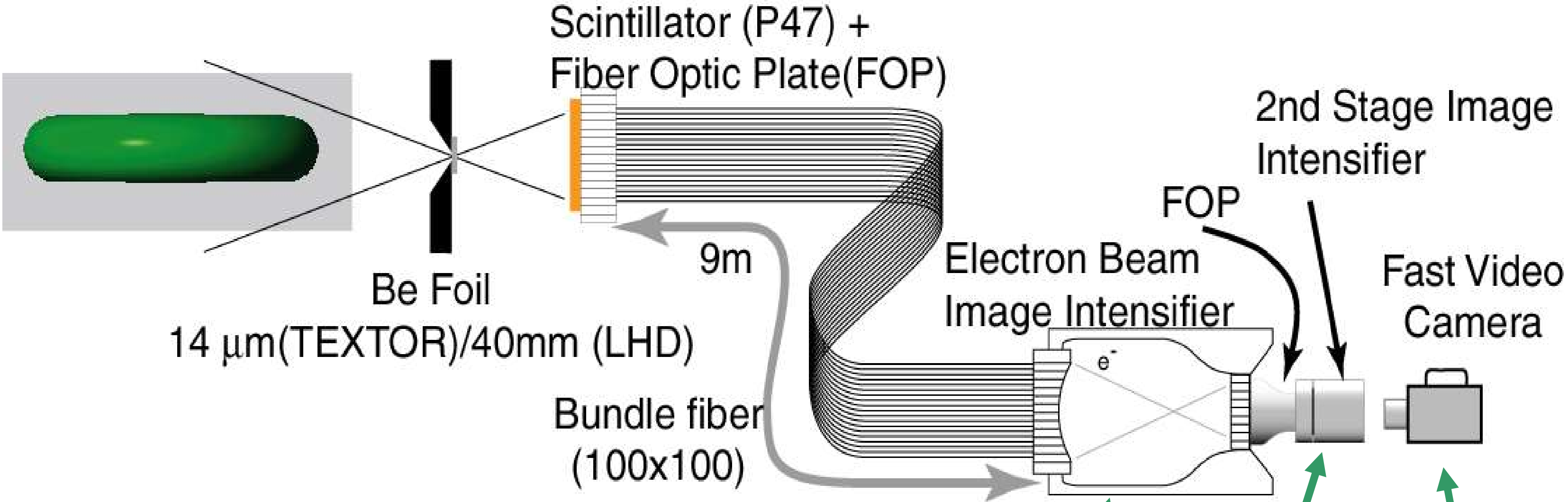
    (TEXTOR, SXII + NTSC Camera, G. Fuchs et. al., EPS 23J(1999)757)

    (LHD, TEXTOR, SXII + Fast Camera, S. Ohdachi et. al., RSI vol.74(2003)2136)
  - PPPL (NSTX, SXII + Fast CCD on-chip memory, R. Feder, B. Stratton et. al.)

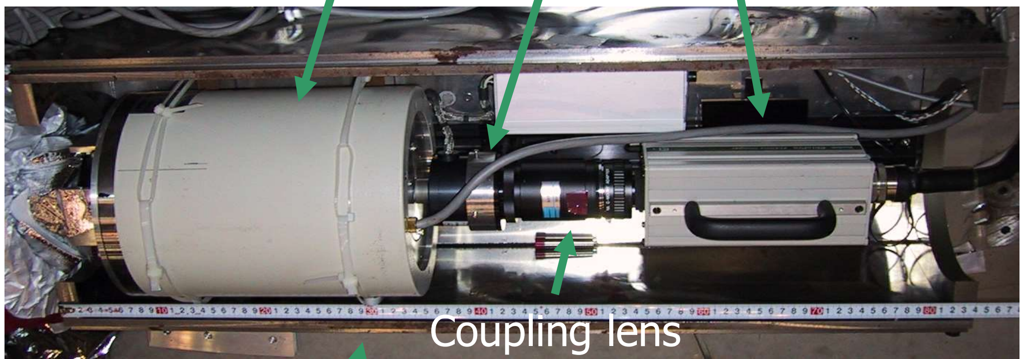
Spatial structure of the MHD fluctuation has been measured with our camera system.

→ Interpretation of the images are needed now.

Hardware of the camera system

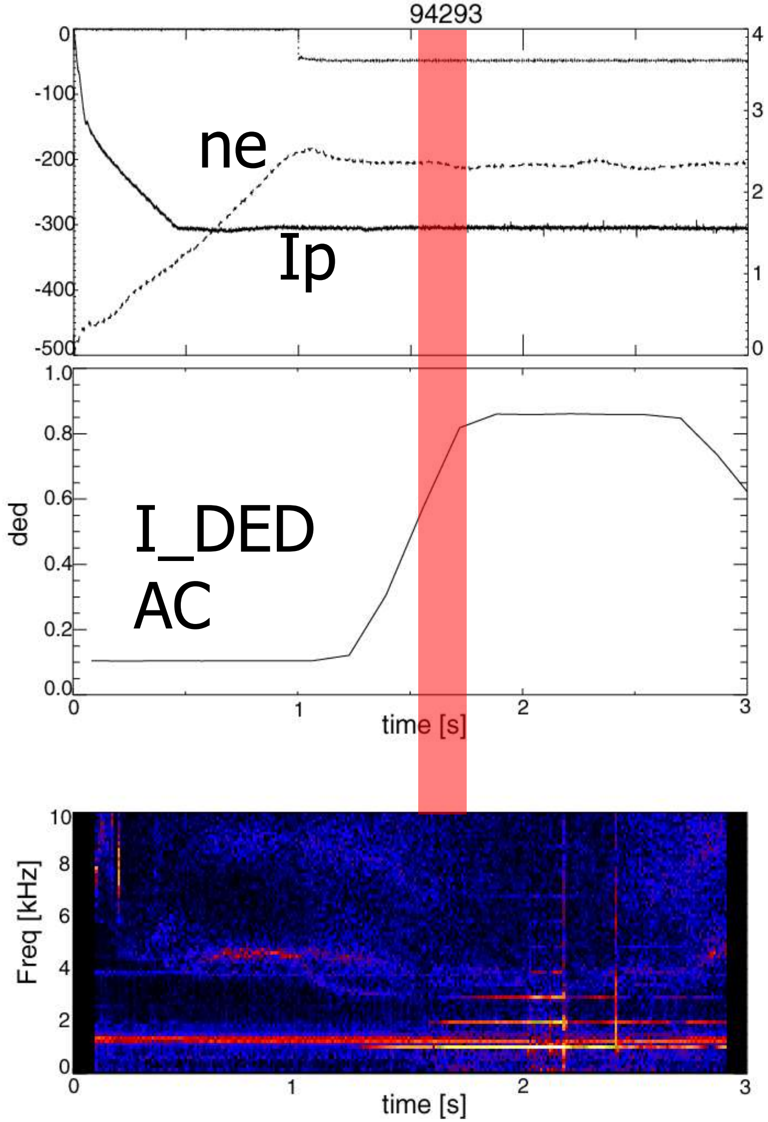


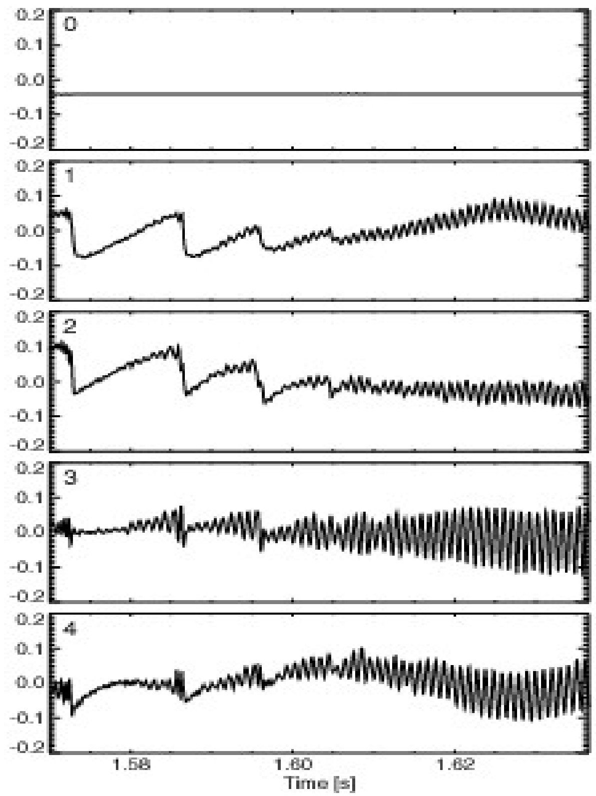
- Fast video camera
  - KODAK:4540MX / (Vision Research Phantom)
  - 30fps-4500fps(256x256)
  - 13500fps(128x128)
- Fluctuation measurement is realized from fast optical system with large diameter scintillator screen(10cm).

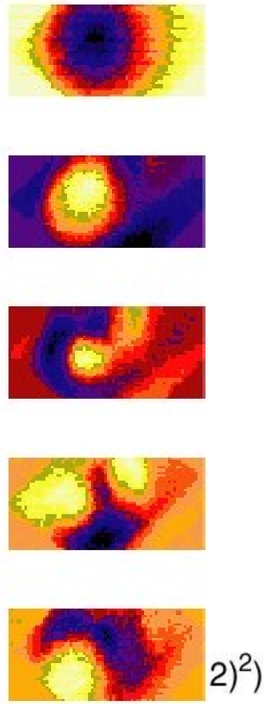


Iron magnetic shied 2.5cm in thickness

Sample Data - m=2 island





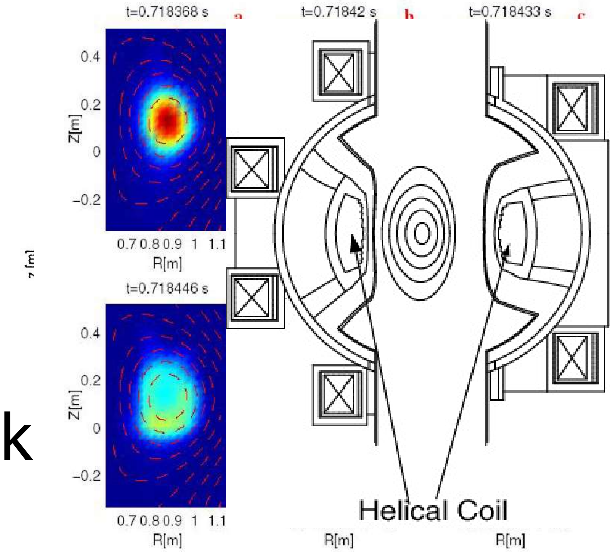


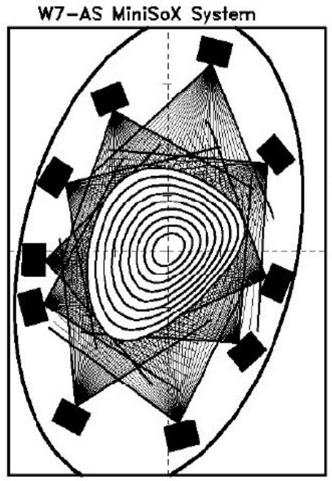
- When m/n = 3/1 perturbation field is applied in TEXTOR tokamak, generation of the rotating m=2 magnetic island is observed.

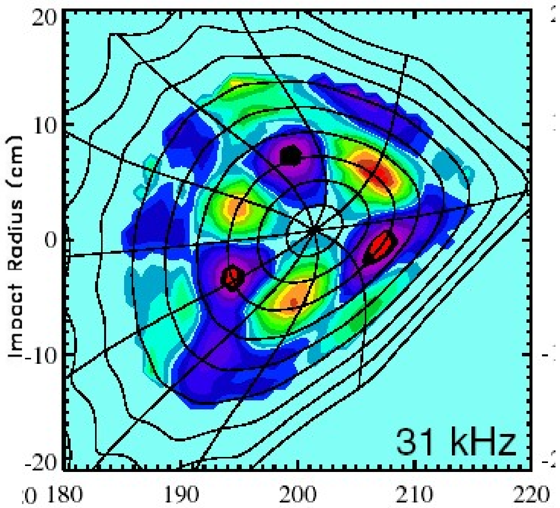
Poloidal Tomography System

TCV

Tokamak







- Many important physics are studied with these system having a good spatial resolutions.
- Many detectors (~2 x m) surrounding plasma are needed for good reconstruction.
- Due to the neutron flux onto the detectors, they can not be used in larger devices. In Large Helical Device, e.g. , such a configuration can not be realized with large helical coil system.
- Installation of tangentially viewing camera is much easier.

Merits / Draw-backs

- Simpler hardware than those in poloidal tomography system.
- We can make use of the human insight via pattern recognition of the structure of the fluctuations.

- ✘ Dynamic range (14bit/~10bit) and framing rate (300kHz/~20kHz) of the present system is not as good as poloidal array system.
- ✘ Difficulty in reconstruction.

- Radiation profile at a poloidal cross section is needed when we want to compare with the theories.
- With reconstructed images, we can study two dimensional effect;
  - e.g. magnetic island shape/size?
  - e.g. ST (Where does the reconstruction take place?)
  - e.g. Ballooning-like nature of the MHD activities.

2D Tomography methods

- It is better to make the full use of rich experience in 2D reconstruction in fusion plasma, since it works under difficult conditions.

- Series expansion methods (Fourie-Bessel expansion, Cormack, ..)
  - shape of the flux surfaces are assumed
- Matrix based methods (ill-posed problem; regularization is needed)
  - Radiation profile using arbitrary grids can be reconstructed; we can estimate the shape of the flux surfaces (W7-AS).

$$g = Af,$$

$$g = \{g_1, g_2, \cdots g_N\} \ : \ \text{Measured data}$$

$$f = \{f_1, f_2, \cdots f_M\} \ : \ \text{Local Emissivity}$$

minimization of $\Sigma(|g - Af|^2) + O(f)$,

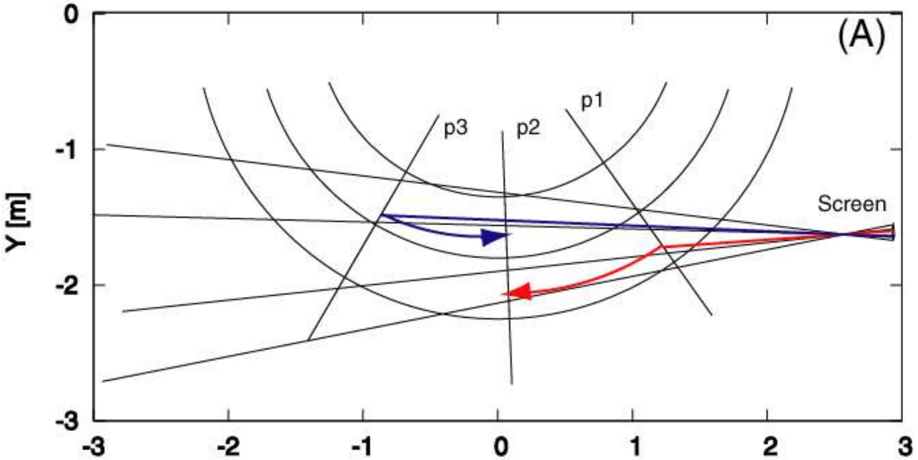
$O(f) = \Sigma\, |f|^2$: Tikhonov-Phillips normalization

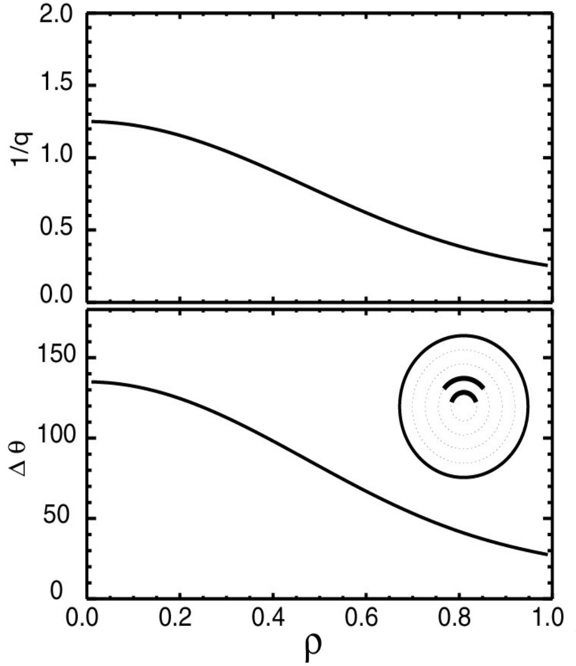
$O(f) = \Sigma\, |\nabla f|^2$: SVD based method by Iwama

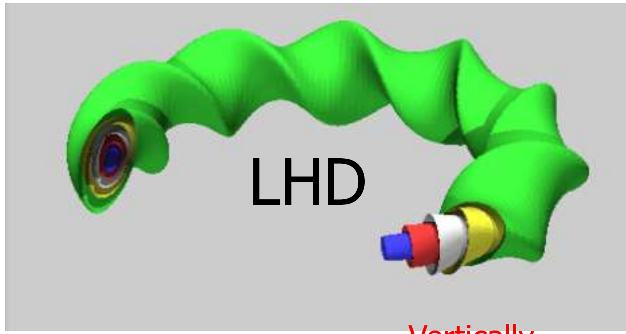
$O(f) = -\Sigma(f \log(f))$: Maximum entoropy method

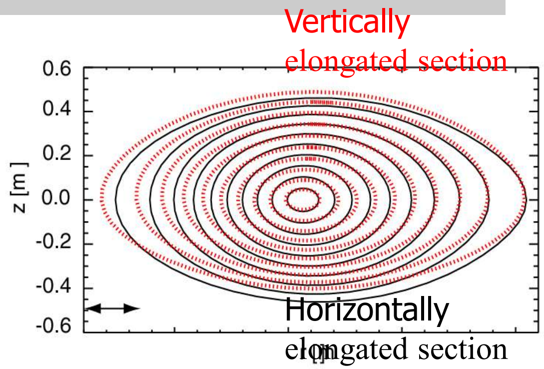
Toroidal /Helical Asymmetry

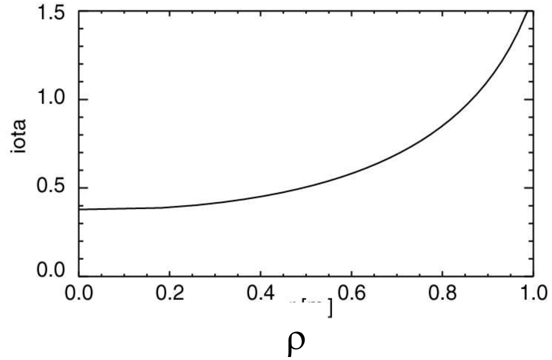
TEXTOR(circular tokamak)







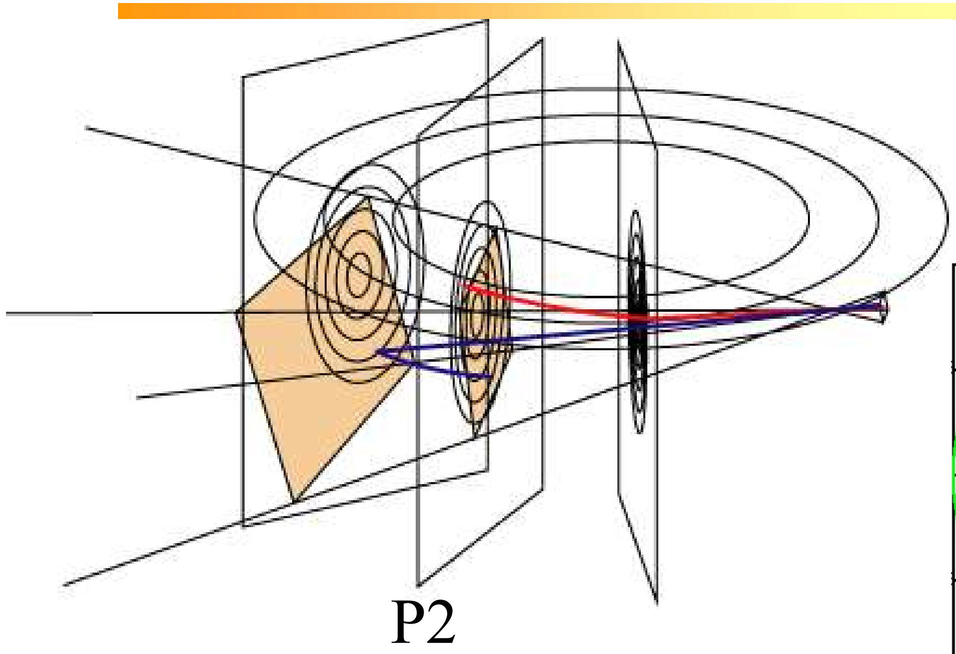




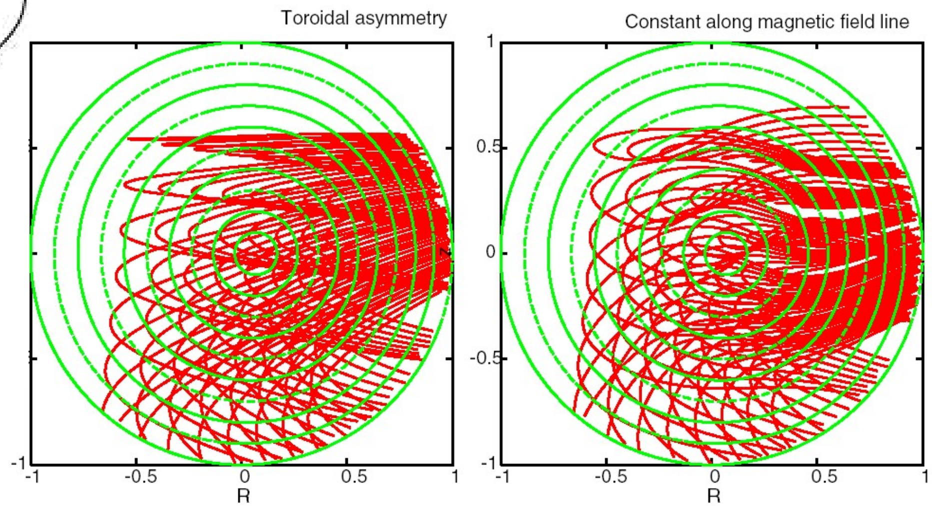
- If we assume toroidal symmetry, information about the poloidal asymmetry is also lost from finite rotational transform.
- In addition to tokamak, in Helical device, we need to assume rotation axis. Radial resolution is worse if we rotate geometrically rotate the flux surface. → not so useful

iota=5(l=10) geometry

Constant radiation along field lines



Equivalent line of sight



- It is not possible to reconstruct 3-dimensional structure from only one projection. If we assume symmetry, 3D reconstruction problem can be reduced to the 2D problem.
- In order to analyze structure at the MHD phenomena, constant radiation along magnetic field lines might be good.
- We need to know the equilibrium magnetic field.

Tomographic Inversion

$$\boldsymbol{S} = L\boldsymbol{E} + \boldsymbol{e}.$$

$\boldsymbol{S}$: Measured signals

$\boldsymbol{E}$: Radiation profile at the reference polidail plane(P2)

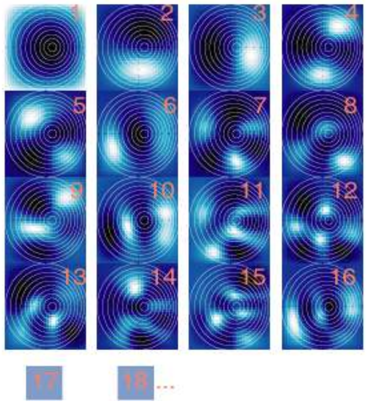
$\boldsymbol{e}$: Residual errors.

$$Q = \gamma \sum |C\boldsymbol{E}|^2 + \frac{1}{M} \sum |\boldsymbol{S} - L\boldsymbol{E}|^2$$

Here, C is the laplacian operator.

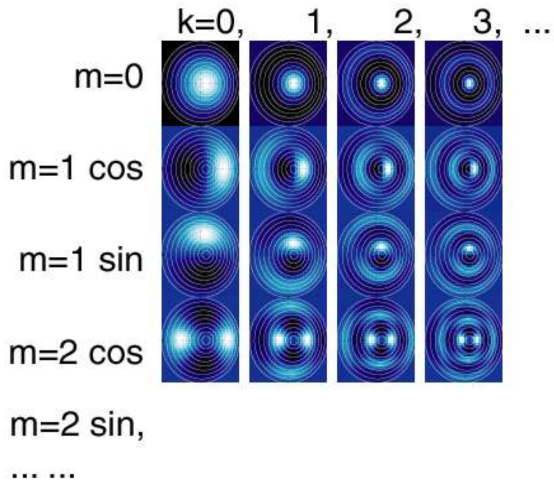
When C^{-1} can be decomposed as UWV^t,

$$\boldsymbol{E}(\gamma) = \sum_{j=1}^{p} w_j(\gamma) \frac{\boldsymbol{u}_j \cdot S}{\sigma_j} (C^{-1}\boldsymbol{v}_j)$$



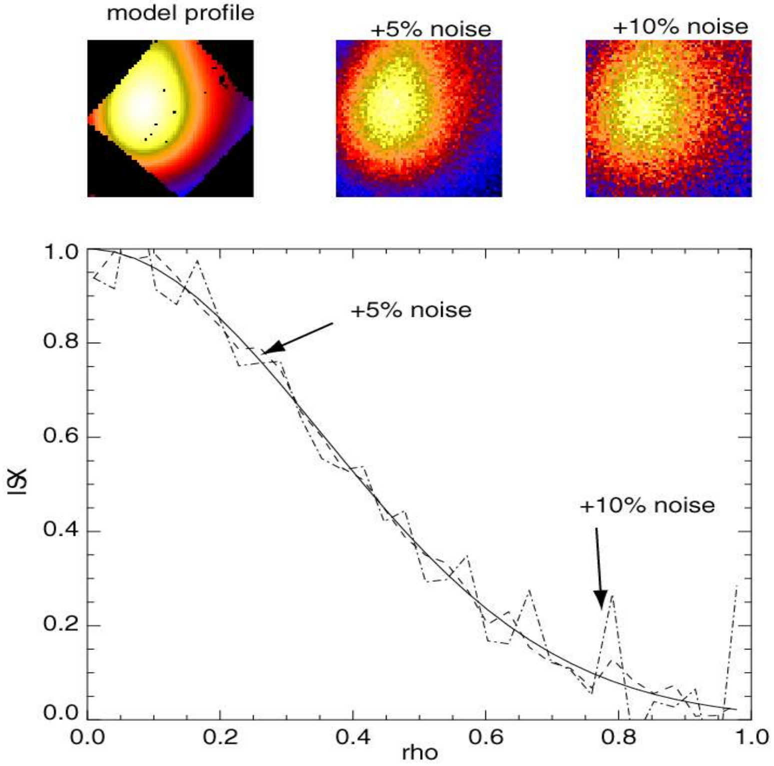
$$\sum_{m=0}^{\infty} \sum_{l=0}^{\infty} a_{mk} \exp(im\theta) J_m(\lambda_m^l \rho) a_{mk}$$

a_{mk} is determined by the least square fitting.

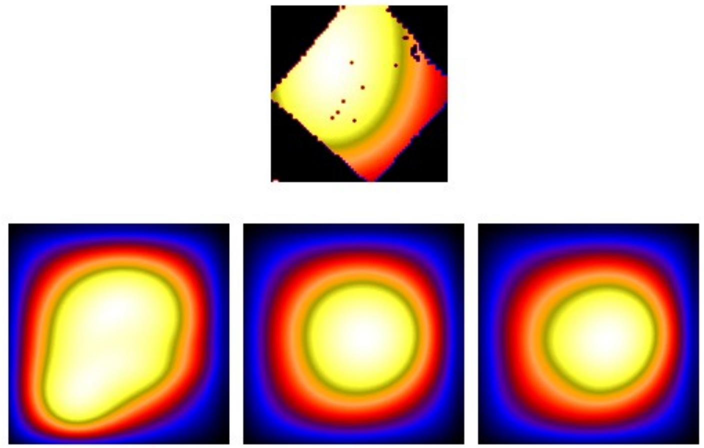


- Since the matrix is very large(e.g.2500x1024), it takes so long cpu time in non-linear optimization, e.g. ME method.
- For matrix based method, we make use of Iwama's method. For series expansion, we use Fourie-Bessel.

Difficulty in reconstruction



test image assuming poloidally symmetric profile



Magnetic Axis -5cm 0cm +5cm

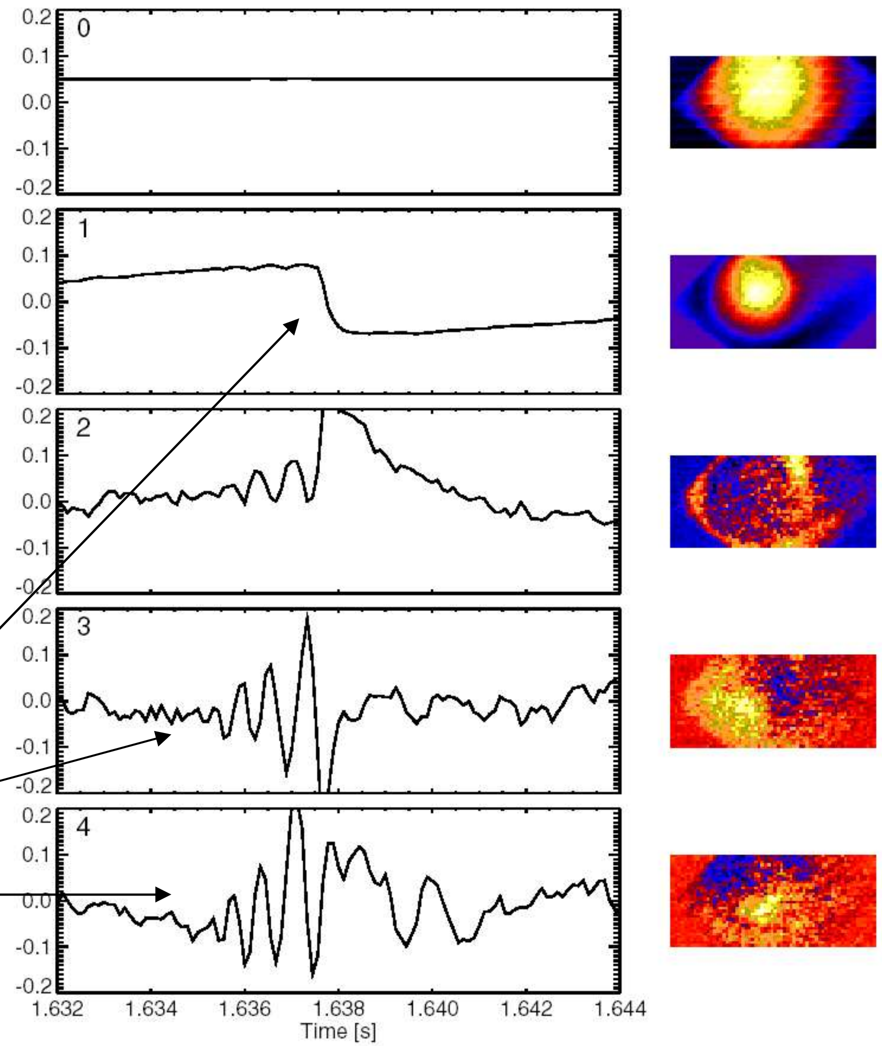
- Equivalent line of sight covers whole plasma cross section.
- Adding white noise is not serious problem.

- In 2D tomography, careful adjustment is required for good reconstruction. Unfortunately, equivalent line of sight in 3D geometry is easily moved.

- Determination of the equilibrium should be done using other diagnostics.

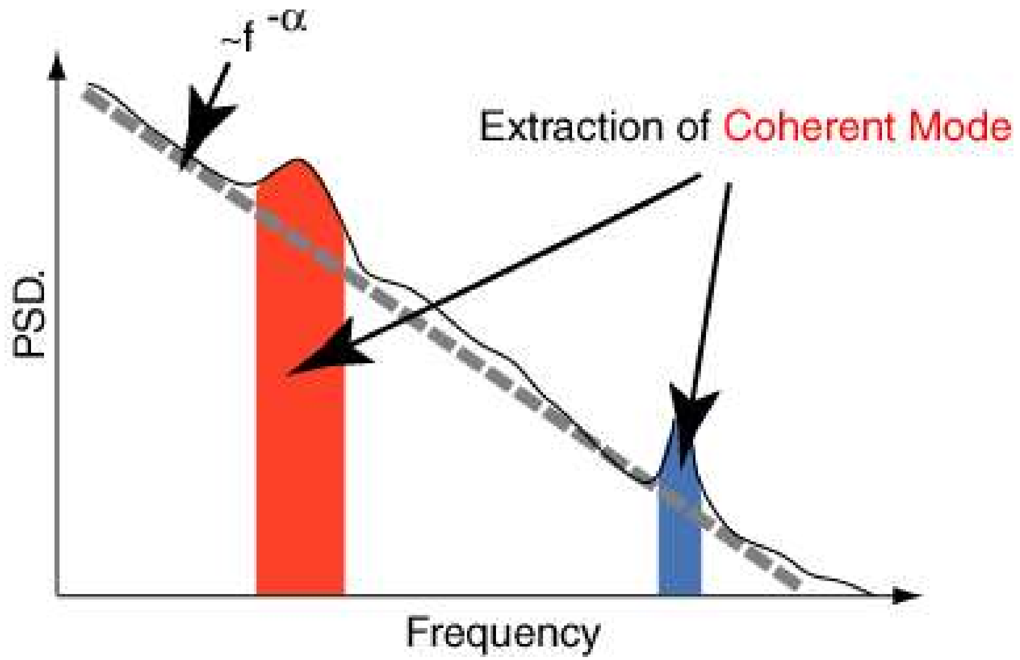
SVD to 2D data/extraction of fluc. components

- There are so many information in 2D moving pictures. However, it is difficult to retrieve information only by watching at the video data.
- In multi-channel data, normal graph can help understanding.
- SV decompose works as tool for digest information. They are also used after tomographic study in W7-AS/TCV after making reconstructions.
- With SVD, sawtooth crash and m=1 precursors can be separated well.



FFT for analysis for fluctuations

Fourier Spectrum Obtained by FFT



- Fast Fourier Transform (FFT) is used for two purposes in fluctuation study.
- Estimate for spectrum shapes. (e.g. comparison with turbulence theory)
- Extraction of coherent modes. (e.g. comparison with MHD instabilities.) For this purpose, the singular value decomposition (SVD) method has advantages over FFT method.

Singular Value Decomposition

$$A = U\,W\,V^t$$

Space structure(topos)　　Time evolution(chronos)

$$\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{21} & a_{21} & \cdots & a_{m2} \\ & \cdots & & \\ & \cdots & & \\ a_{n1} & a_{n1} & \cdots & a_{mn} \end{pmatrix} = \begin{pmatrix} u_{11} & u_{21} & \cdots & a_{m1} \\ u_{21} & u_{21} & \cdots & a_{m2} \\ & \cdots & & \\ & \cdots & & \\ u_{n1} & u_{n1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ & \cdots & & \\ 0 & 0 & \cdots & w_m \end{pmatrix} \begin{pmatrix} v_{11} & v_{21} & \cdots & v_{m1} \\ v_{21} & v_{21} & \cdots & v_{m2} \\ & \cdots & & \\ v_{m1} & v_{m1} & \cdots & v_{mm} \end{pmatrix}$$

By SVD, a matrix A (M $\times$ N) made up of N time series of M frames ($\boldsymbol{a}_m = (\boldsymbol{a}_{m1}, \boldsymbol{a}_{m2}, \cdots, \boldsymbol{a}_{mn})$) is decomposed into three matrices, U (M $\times$ N), V (N $\times$ N) and a diagonal matrix W(N $\times$ N); $A = UWV^t$. The columns of U and V are spatial and temporal eigenvectors and are called topos and chronos, respectively. A time series $\boldsymbol{a}_i$ can be written by a combination of independent components of t he topos($U = (\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_n)$) and the chronos as,

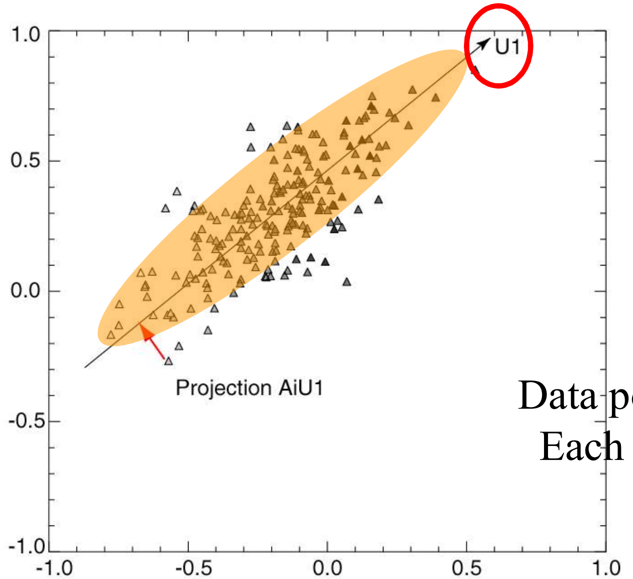
$$\boldsymbol{a}_i = w_1 \times v_{i1} \times \boldsymbol{u}_1 + w_2 \times v_{i2} \times \boldsymbol{u}_2 + \cdots + w_m \times v_{im} \times \boldsymbol{u}_m$$

Measure of the contributions from each orthogonal functions

- Spatial and temporal components are obtained simultaneously.
- Large few components can explain the characteristics of the fluctuations usually. (summary of the data)

Advantage of SVD method

SVD is equivalent to the
method called
PCA(statistics)
EOF(meteorology)
POD(Fluid dynamics)

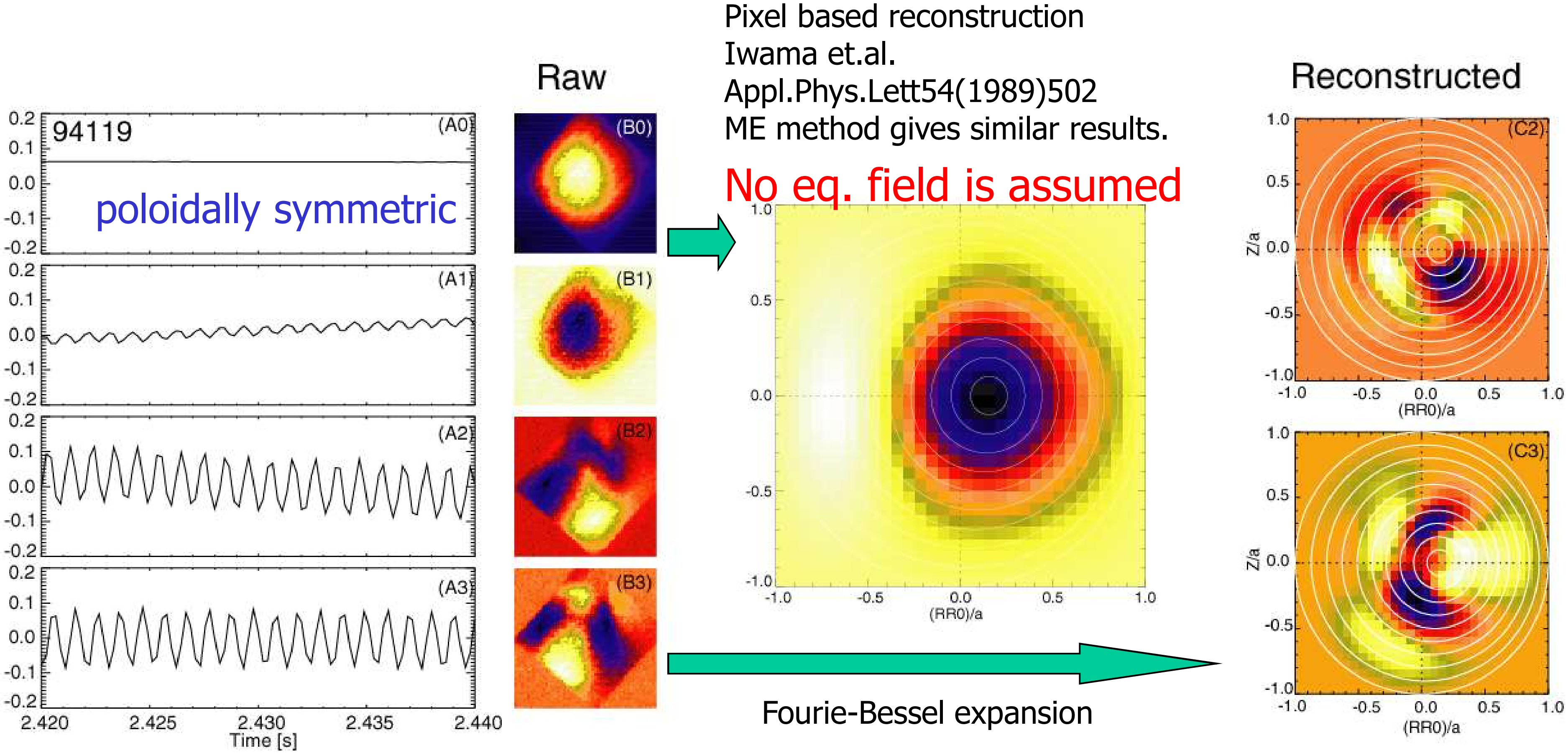


Complicated data are summarized by selecting a proper unit basis vector.

Data point:
Each trial of measurements

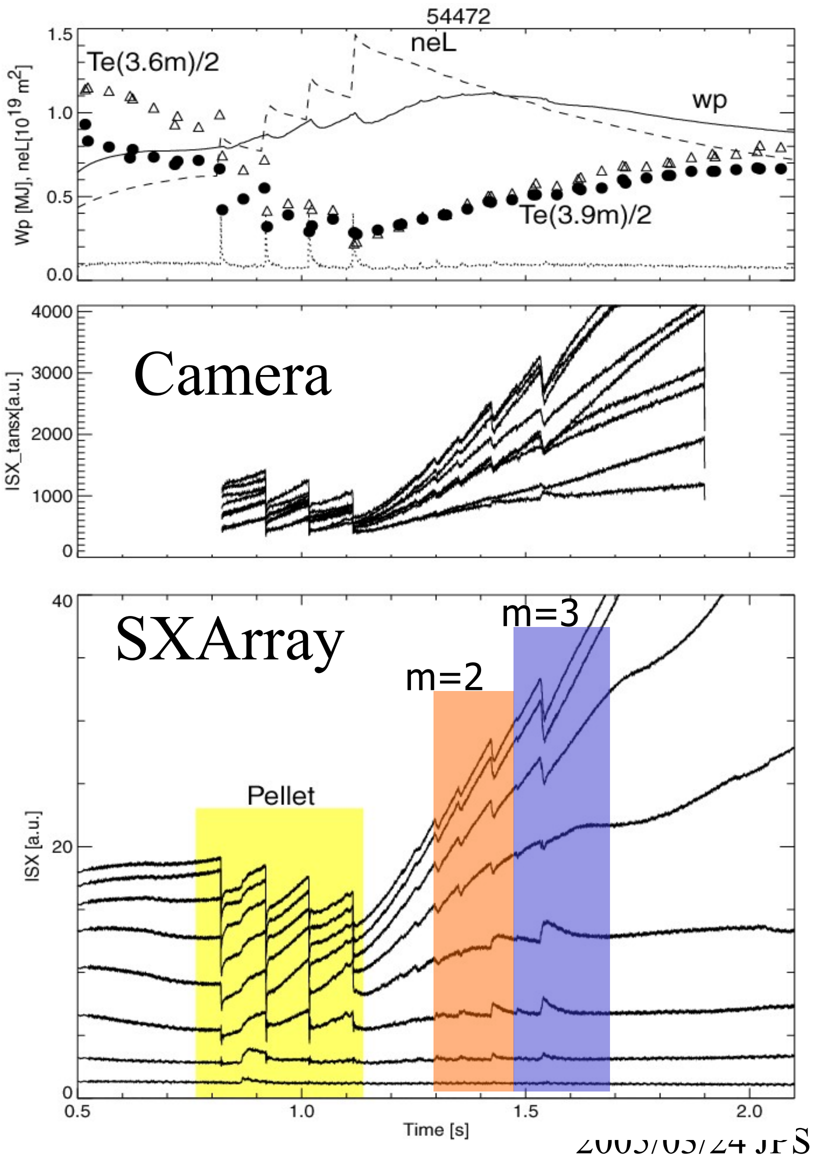
- Topos (space structure) u_i is unit basis vectors in n-dimensional system that maximizes the summation of the projection of the measured data $\Sigma u_i a_i$. In this process we can make use of the similarities or correlations of the multi-channel data.
- Chronos (time evolution) can be arbitrary orthogonal functions. It is useful when the frequency of the fluctuations is changing or when the events with step-function like waveform, while, in FFT, fixed-frequency trigonometric functions are used for the basis of the decomposition.

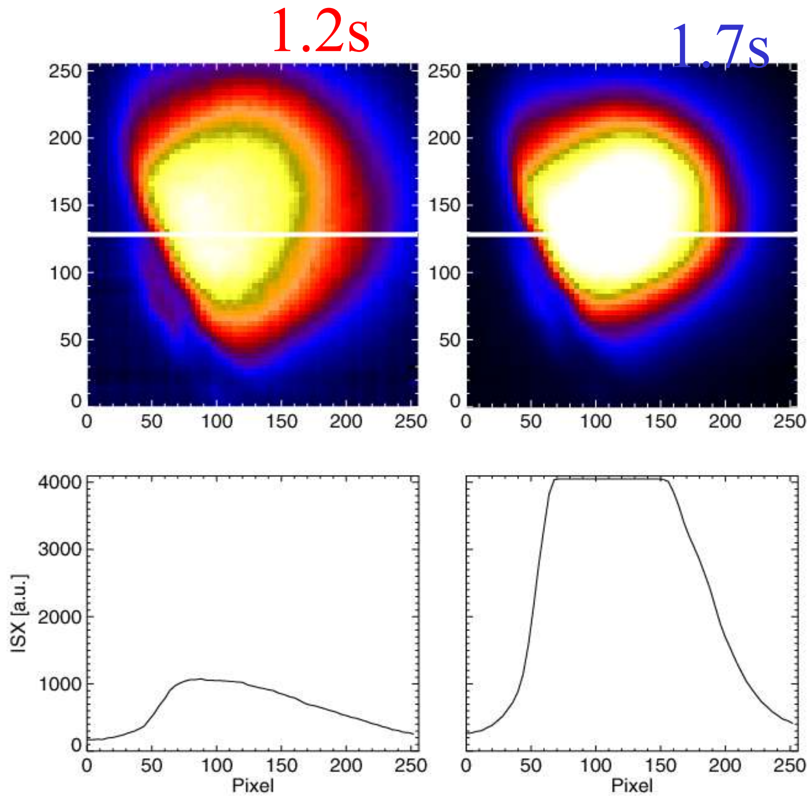
m=2 islands and its reconstruction



- From stationary component(B1) we determine the Shafranov-shift.
- With obtained shift and the q-profile, equilibrium fields are composed.
- C2 and C3 are reconstructed from B2 and B3 by fourie-bessel methods.

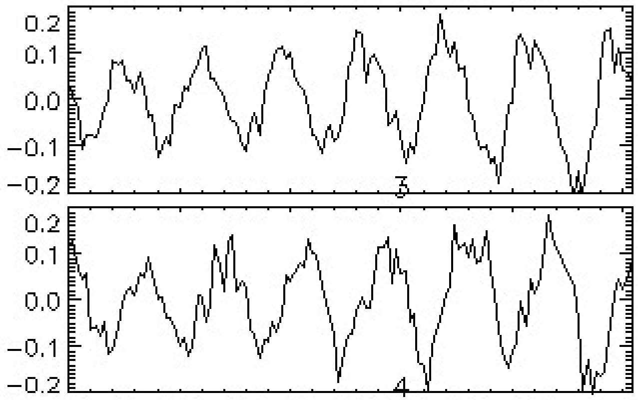
Fluctuation structures observed in LHD

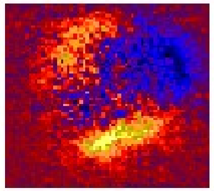


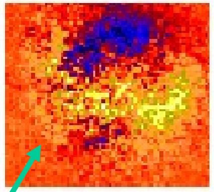


- Internal minor disruptions are observed in fairy low shear and peaked plasmas.
- m=2 postcursor and m=3 pre/postcursor are observed.

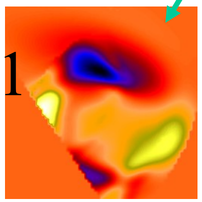
m=2 pre-cursor case

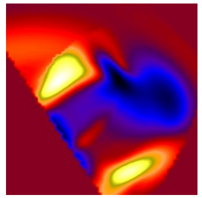






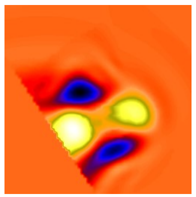
m=2, ρ~0.7±0.1

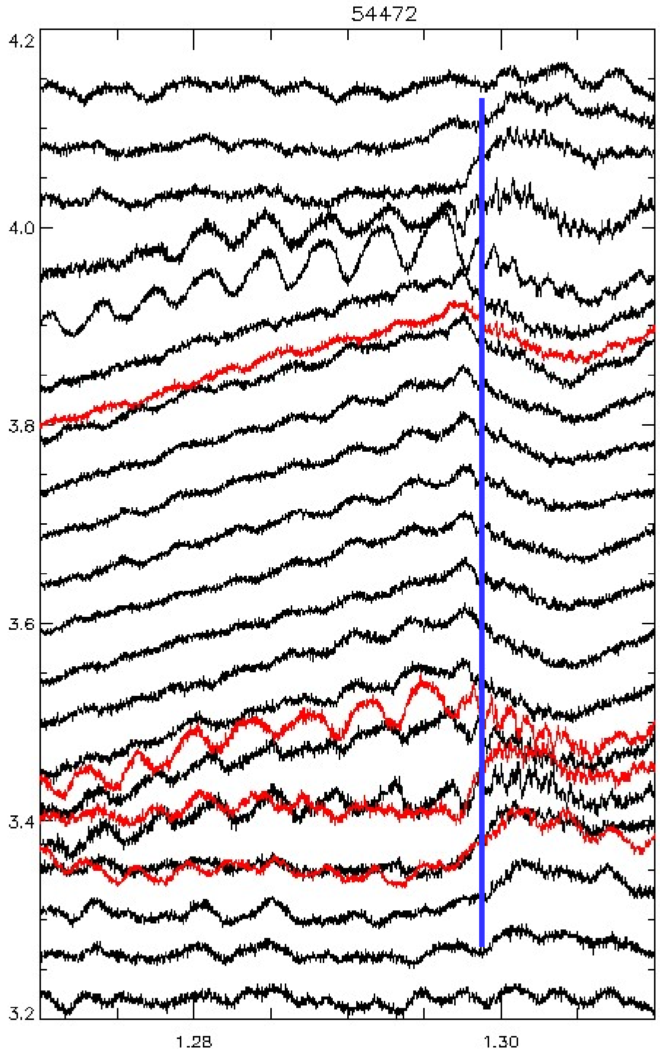




- m=2 precursor. Strange shape is caused by the geometric effect.

m=2, ρ~0.4±0.1



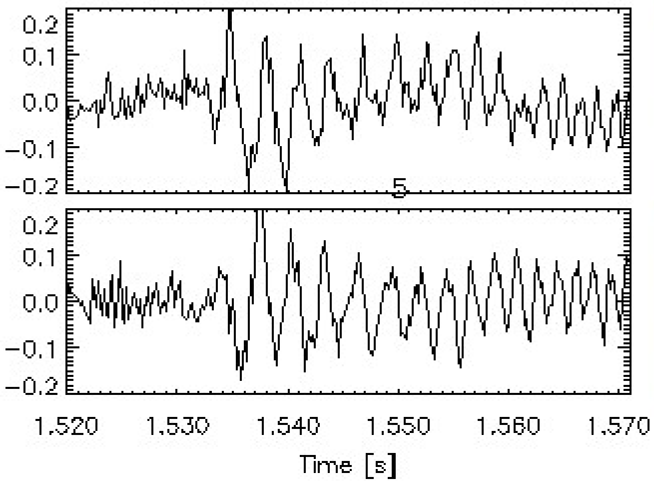


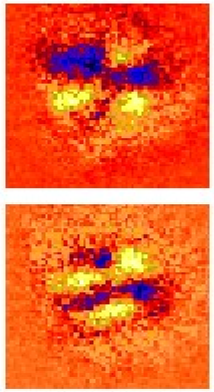
3.5U

6.5U

m=2/n=2?

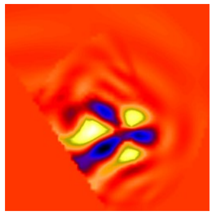
m=3 (with post-cursor)

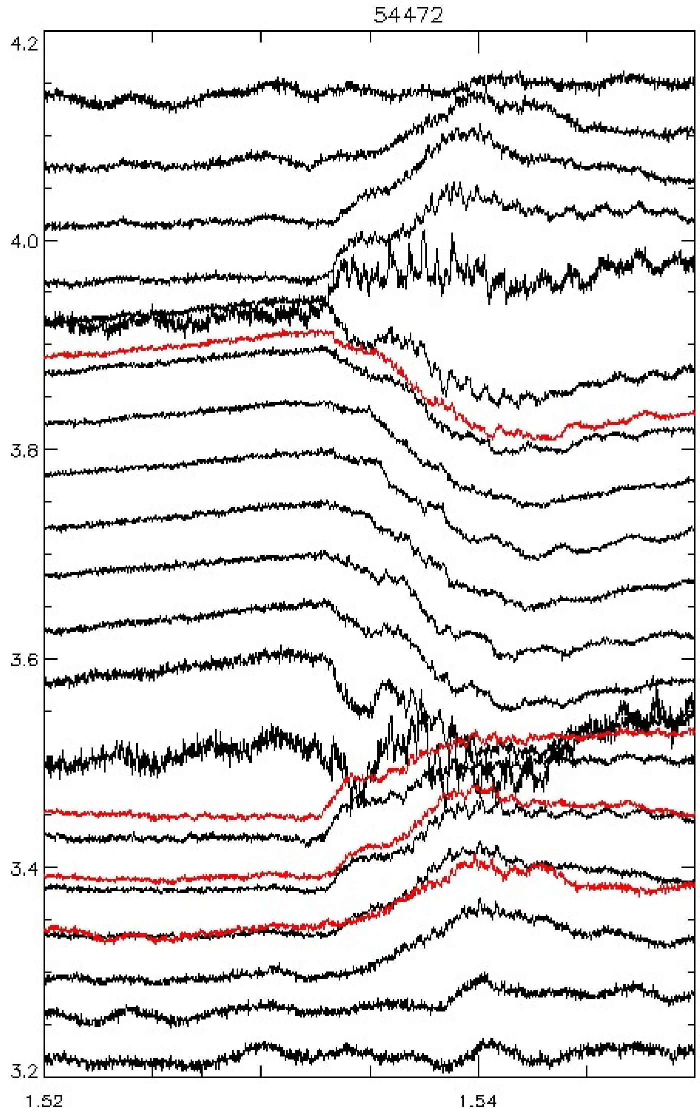




Simulated image

m=3, ρ~0.2±0.1





- At the present, we can not do reconstruction well in helical plasmas. We compare the image with simulated images assuming a symmetric structure.

Summary of the reconstruction

	Asymmetry	Present status	Methods
Tokamak	Toroidal	OK	Pixel based method, ME, PT, SVD(Iwama)
	Const. along Field Lines	OK (When Estimation of the equilibrium is good)	Fourier-Bessel expansion
Helical	Helical	does not work	
	Const. on a flux surface	OK	PT, SVD
	Const. along Field Lines	not yet	Compared to Fourier-Bessel model

Summary

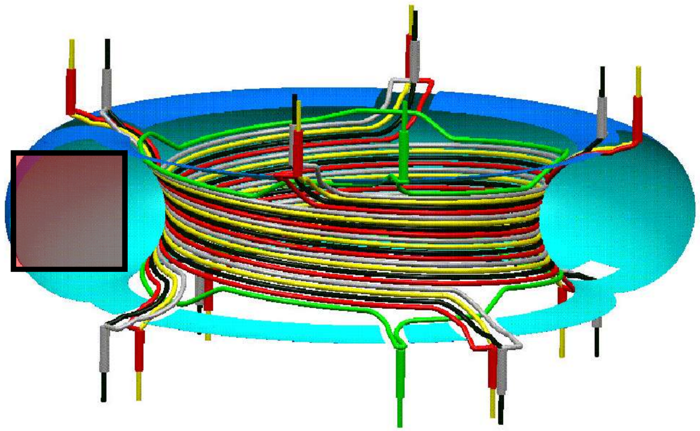
- Tangential imaging using SX radiation is a effective tool to detect moving structure. It is even useful for high m case; we have observed fluctuations having m=3 structure so far.

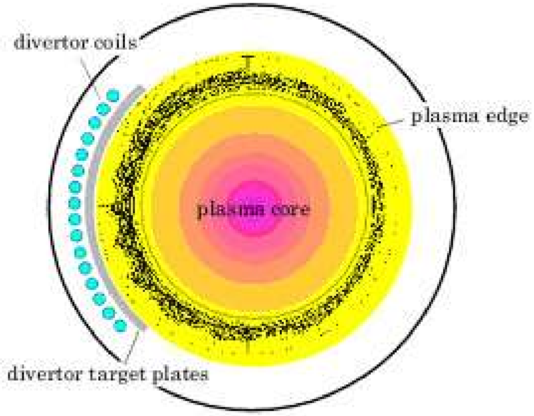
- Tomographic reconstruction in 3D case is more difficult than in 2D case, since the line of sight can not be fixed; it depends on the equilibrium magnetic field.
- When we estimate the equilibrium field well, we can make use of the techniques which is used in 2D tomography. If we can put several detectors, the estimation of the equilibrium can be better.

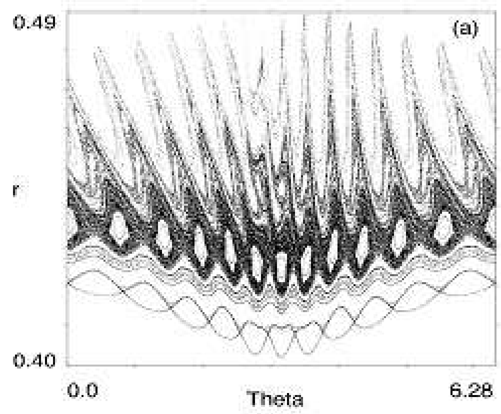
Future plan

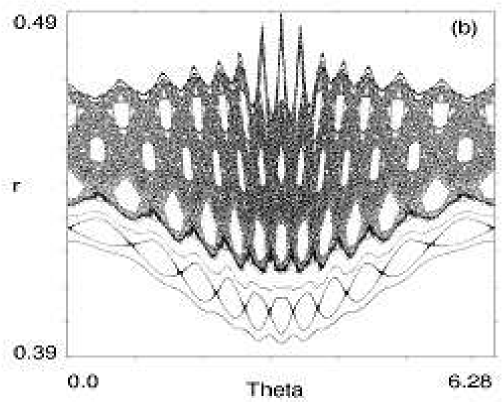
- Reconstruction code should be extended for non-toroidally symmetric case, e.g., LHD.
- Codes based on the flux coordinate system to share code in different machines(TEXTOR, LHD, NSTX).
- Make estimate of the error in the reconstruction processes.

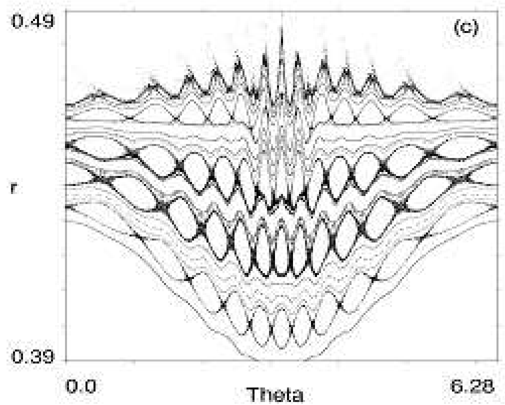
DED coils and poincaré map





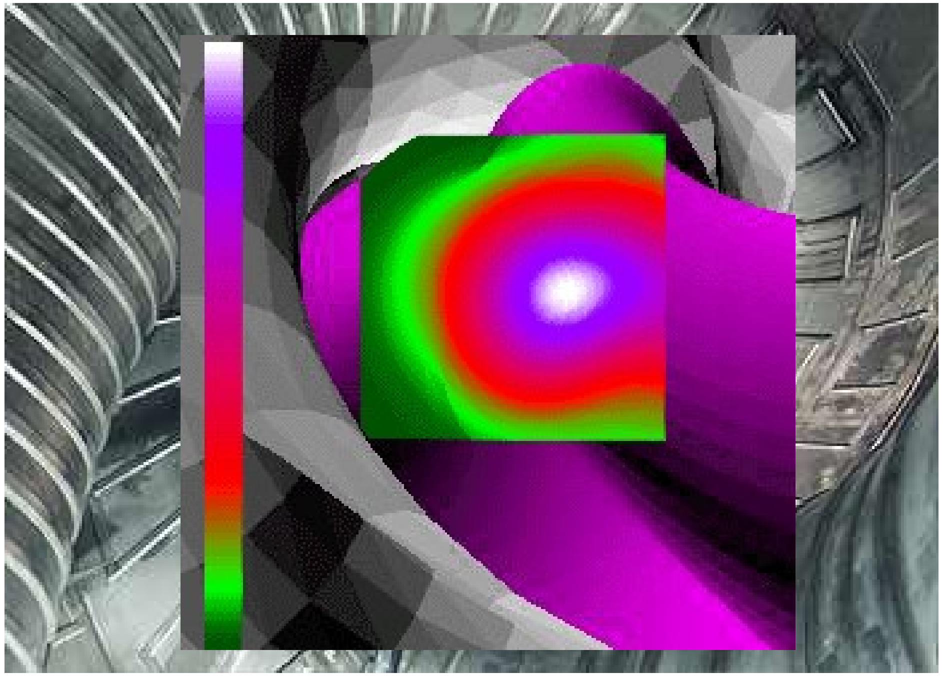


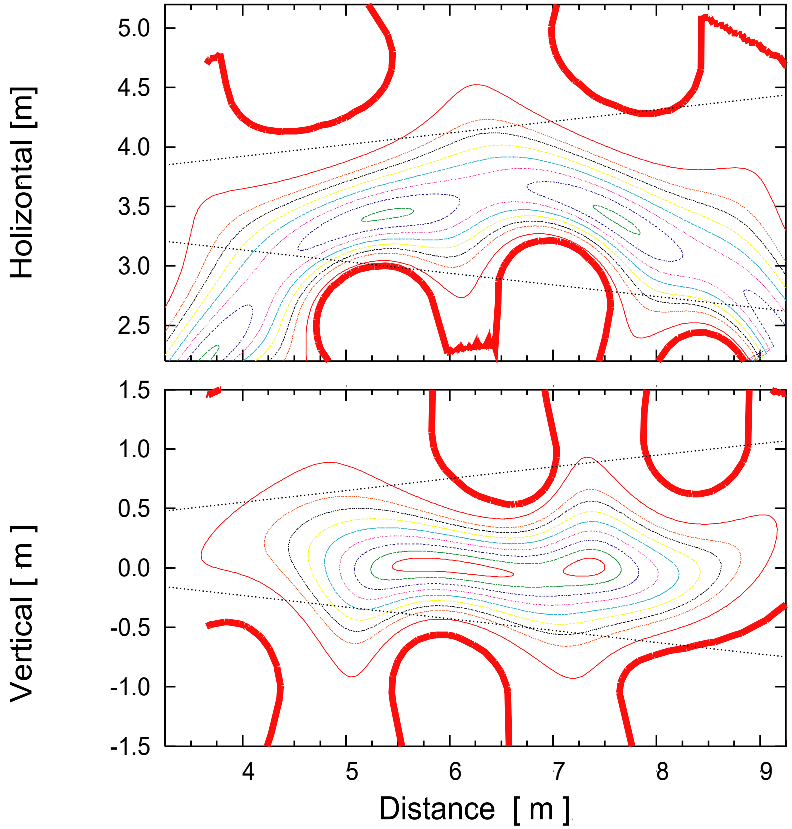




- possible configuration m/n = 12/4, 6/2, 3/1

Measuring area of LHD





SX array and images